Ex-Post Price Impact Modeling
Challenges and Opportunities

INTRODUCTION

Shortly after the market opening on August 1st 2012, a single server owned by Knight Capital flooded the market with persistent buy and sell trades as it attempted to fill 212 small customer orders in 154 U.S. stocks. According to the SEC press release,¹ the surge of trading activity caused by a faulty code deployment resulted in execution of 4 million child orders for almost 400 million shares during the first 45 minutes after the market opening. For 75 of those stocks, Knight’s executions exceeded 20% of the trading volume and contributed to price moves over 500 basis points (bps). For 37 of those stocks, the price moved by more than 10 percent, as Knight’s executions constituted more than 50% of the trading volume. The price impact of Knight’s trades resulted in large unwanted long and short positions and an estimated $461 million (mln), loss as Knight attempted to close these positions. As a result, Knight was forced to seek funding from external investors to stay afloat.

This accident, labeled “Knightmare” in the popular press, was triggered by millions of executions that were never intended to happen in the first place. But accounting for, and minimizing the losses due to the price impact of large trades remains an integral part of the daily cost management routine by institutional portfolio managers and trading desks. According to the broker group data in the ITG Peer Database², the estimated daily impact cost suffered by U.S. institutional investors amounts to more than 15 mln. U.S. dollars [see Table 1]. To put this figure in perspective, with more than 60 clients trading on a randomly selected day, this is roughly equivalent to daily trading losses of $240,000 experienced by a representative member of the ITG Peer Database.

Needless to say, a large fraction of own price impact cost born by non-discretionary traders is unavoidable, as it merely stems from large institutional order sizes or relatively thin liquidity available in traded names on a particular day. On the other hand, the excessive price impact originating from aggressively placed orders traded in temporarily difficult market conditions or during the episodes of high volatility can be reduced, eliminated, or even reversed with a more flexibly scheduled trading strategy. One of the overarching goals of TCA is to identify opportunities for


² The ITG Peer Database is a proprietary database of historical institutional trading activity for the ITG Peer Universe, covering more than 200 of the largest global buy-side institutions.
transaction cost savings without compromising clients’ investment objectives and other constraints such as the time and price limits. Some of the tools developed by ITG for modeling and management of price impact will be highlighted in this short article.3

1. THE CONCEPT OF PRICE IMPACT: AN EXAMPLE

Consider a large order to buy 44,700 shares of Argo Group International Holdings Ltd. [ticker AGII]4 that was filled by a buy-side institution on August 29, 2013. The daily volatility and volume on that day were close to normal, but the trading activity was spotty, and the order was large, representing about 66% of median daily volume for this relatively illiquid stock. The trader received this order shortly after 9:45AM and managed to buy more than 37,000 shares, using a sequence of three mid-point crosses in combination with small trades on lit markets before noontime. The remainder of the order was filled in the lit market, using a balanced mixture of passive [limit order] and aggressive [market order] executions, mostly filled during the last 20 minutes before the market close [Figure 1].

FIGURE 1:
Realized intraday volumes, client’s own participation in total volume, and historical 20th, 50th, and 80th percentiles of the intraday volume distribution
Buy 44700 shares of ticker AGII [66% of MDV] on August 29, 2013

TABLE 1:
Estimated Total Price Impact Cost for U.S. Institutional Investors
[Based on ITG Peer Database common stock trades in 2012Q2–2013Q2]

<table>
<thead>
<tr>
<th>Number of observations (thousands)</th>
<th>Trading volume (USD bln.)</th>
<th>Own impact cost per day (basis points)</th>
<th>Own impact cost per day (USD mln.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Common Stocks</td>
<td>3,072</td>
<td>3.732</td>
<td>13.5</td>
</tr>
<tr>
<td>Very Liquid</td>
<td>1,095</td>
<td>2.916</td>
<td>11.7</td>
</tr>
<tr>
<td>Liquid</td>
<td>737</td>
<td>597</td>
<td>17.9</td>
</tr>
<tr>
<td>Medium Liquid</td>
<td>898</td>
<td>203</td>
<td>22.4</td>
</tr>
<tr>
<td>Less Liquid</td>
<td>342</td>
<td>16</td>
<td>21.5</td>
</tr>
</tbody>
</table>

3 The ideas discussed in this article have been implemented in ITG’s Smart Cost Estimator for post-trade analysis and effectively used for high touch execution consulting.

4 AGII is the common stock of a Bermuda-registered specialty insurance and reinsurance underwriter.
The realized execution schedule for the order (shown by red bars in Figure 1) was dominated by dark executions around 11:00AM and 12:00PM, interspersed by mostly small trades in other time periods. Except for the episodes of higher than normal trading activity between 10:25AM and 11:10AM and shortly before market closing, trading volume for this stock remained lower than normal during the day.

Even a casual market observer would have little doubt that filling such a large buy order for any thinly traded stock would be associated with an upward price drift, unless there is a countervailing selling pressure from other market participants. That was not the case in our example, as indicated by the observed price path of the stock during the day (bold black curve on Figure 2), which rallied strongly on the back of favorable macroeconomic reports in the morning, reached a plateau around 11:00AM, and stayed relatively flat for the rest of the day, exhibiting only a slight downward drift and concluding the day with a strong but short-lived rally.

With the benefit of hindsight, one might attempt to answer the question: Was the execution schedule shown on Figure 1 justified? One may wonder, for instance, whether a larger fraction of the order could be filled earlier on that day when the

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**FIGURE 2:**
Mid-quote price paths for the realized and hypothetical [VWAP-style] trading strategies, along with the expected price path in zero trading scenario
Buy 44700 shares of ticker AGII (66% of MDV) on August 29, 2013

Price Path Comparison: Dark Pool Trading in Realized Strategy Only

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Source: ITG
price was still relatively low, without searching for dark liquidity. Or, alternatively, would delay of own trading until late afternoon hours result in a better average execution price?

2. PRICE IMPACT, EX ANTE AND EX POST: WHAT IS THE DIFFERENCE?

It is important to distinguish between the ex post [post-trade] and the ex ante [pre-trade] price impacts. We believe there is a big difference between the two in our ability to model and estimate these measures. While the pre-trade price impact is a function of the order size and stock characteristics, as well as the market conditions that have been observed before the order arrival time, this function does not account for the observed market conditions during the lifetime of the order, i.e. between the arrival time and the time of the last fill. The post-trade price impact takes those conditions into account. For example,

- Once high volume has been observed, and it is determined that most of that volume is due to other participants, on average, the own price impact is much smaller than its pre-trade expectation;

- Once favorable market momentum or alpha has been realized, we know that the own price impact is smaller than its pre-trade expectation, since we participate in trades predominantly as a liquidity provider [not a liquidity taker];

- Once there is an opportunity to trade in ATFs or other off-exchange liquidity pools [which we call, generically, the “dark pools”], our own price impact for reasonable trade sizes is significantly smaller than its pre-trade expectation.

The ex post price impact for the realized trading strategy is defined as the expected difference between the observed price trajectory and the expected price trajectory that would have been realized under the hypothetical strategy of trading nothing at all. While the ex ante [pre-trade] price impact represents the anticipated effect of own order execution on the price path before trading has even started, the ex post [post-trade] price impact estimate measures the potential effect of our unconditional withdrawal from the market after the trade has been completed, taking into consideration the fact that the state of the market has been observed for the entire execution horizon. Since ex post price impact is based on the richer information set, it gives a more nuanced perspective on own executions, enhancing our understanding of the trading process, highlighting blunders [and achievements] during the execution process, and suggesting the ways to improve trading performance.

Figure 2 illustrates how the trading schedule affects the realized price path of the stock in the example considered above. We take the realized mid-quote price path [shown by the bold black curve on the top plot of Figure 2] as a starting point, and consider how it would change if the realized execution schedule is switched to the front-loaded, neutral, and back-loaded schedules. Specifically, we consider price paths implied by five alternative hypothetical trading schedules created from the historical VWAP strategy and applied from the order arrival time 9:45AM until the end of the day. Strategy VWAP denotes the historical volume-weighted average price strategy. The strategies VWAPvf, VWAPf, VWAPb and VWAPvb denote, respectively, the very front-loaded [vf], front-loaded [f], back-loaded [b] and very back-loaded [vb] VWAP-style strategies. For ease of interpretation, the expected mid-quote price paths and their underlying hypothetical trading schedules are shown in matching colors. As discussed above, the expected price paths reflect the realized market conditions observed before, during, and after the order execution.

5 The strategies are shown at the bottom of Figure 2.
3. THE REALIZED ALPHA AND GENERAL MARKET DRIFT

Our model permits the decomposition of the price movement within any time interval into the price reactions attributed to:

- Own child order executions on lit and dark venues that occur during the current time interval or the carry-over effects from own child orders filled in the past,
- Contemporaneous trading in the same ticker by other market participants, and
- Contemporaneous trading in other markets [such as the trades contributing to the market reaction to the news about the general state of the economy or the news relevant to the reinsurance industry].

The notion of client’s short-term “alpha” introduced and described in greater detail in the recent ITG white paper is closely related to the ex post price impact of the client’s own trading. At the individual order level, the realized alpha is the market-adjusted return that would have been realized if we withdrew from trading in the given interval entirely [the “zero trade” scenario]. Intuitively, the realized alpha for the given order is the part of the interval return that is not attributed to own trading and orthogonal to general market, sector and industry movements; it includes the impact of other participants’ trades and stock-specific information, such as fluctuations in the market demand and supply of liquidity.

Why does this decomposition matter? If we believe that the price hike observed at the onset of own trading was triggered by market participants reacting to external factors largely unrelated to own trading, one could have safely filled a much larger fraction of the order in the lit market without adversely affecting the price of the stock, simply by scheduling more buy trades before the realized price path achieves its peak and ending up with significant cost savings in comparison to the realized execution price path. To determine whether there is any performance improvement potential for the realized execution strategy, we decompose the realized returns in each five-minute interval into the parts attributed to own price impact (which is strategy-dependent) and the price impact of other market participants (which is assumed to be invariant to tweaking of the own strategy), and calculate the implementation shortfall costs for each of those components under the realized and hypothetical strategies.

Figure 3 displays the estimated implementation shortfalls and their decompositions into the components due to own impact [in green], due to alpha [in red], and due to market/sector/industry factors, also known as momentum proxy cost, [in blue] under alternative VWAP-style hypothetical strategies. In spite of having the relatively low cost of 42 bps attributed to own price impact, with a major part of the order being filled in dark pools, the realized trading strategy ended up with a rather substantial total cost of 201 bps, because most of the order was filled after 11AM, after the unfavorable price movement attributed to alpha and momentum proxy movement had been completed. More front-loaded trading strategies [vwap.vf and vwap.f] appear to have lower alpha and momentum proxy cost components in comparison to the realized trading schedule. The moderately front-loaded vwap.f strategy appears to achieve the right balance between alpha and momentum proxy cost savings and the increase of own impact cost, which results in the total cost savings of 15 bps compared to the realized strategy. The savings would be achieved, ex post, even though the executions of orders in dark pools are generally expected to be less expensive, due to their low price impact. In contrast, if the frontloaded trading strategy VWAP.vf were implemented, it would have resulted in the total cost of 210 bps because of the substantial increase of own price impact cost from 42 to 138 bps,

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despite the noticeable alpha cost reduction (56 bps for VWAP.vf vs. 115 bps for the realized strategy) and momentum proxy savings (16 bps for VWAP.vf vs. 44 bps for the realized strategy).

It is important to keep in mind that most of these cost savings would be out of reach, without at least some ability to predict the alpha and/or momentum proxy return dynamics in real time. Assuming that only own price impact returns can be reliably predicted ex ante, the realized strategy would beat any of the alternative VWAP-style strategies considered in this example by at least 3 bps, with the biggest advantage (99 bps) against VWAP.vf. Assuming that the alpha or momentum proxy dynamics have a limited ex ante predictability, one can find the level of predictive skill that would have justified switching from the realized to any of these alternative strategies.

We repeat the above experiment by constructing an alternative set of hypothetical strategies, which trade exactly the same amount as the realized strategy (32600 shares) at the three mid-point crosses, but apply VWAP-style front- or back-loaded trading schedules to the remaining part of the order (14100 shares) filled in the lit markets. Figure 4 shows the results. As one can readily see, the ex post advantage of the front-loaded vwap.f trading schedule relative to the realized trading strategy disappears. However, the very front-loaded vwap.vf strategy appears now to be a better choice despite still having the highest own impact among all the alternative strategies. Also, in contrast to Figure 3, the portion of the cost attributed to own price impact is no longer minimal for the realized trading strategy. In fact, taking advantage of the dark pool liquidity while delaying lit executions until the end of the day, as implied by the very back-loaded VWAP.vb trading strategy, would have resulted in the own impact cost savings of 12 bps relative to the realized strategy and 21 bps relative to the very front-loaded strategy for lit market executions.

However, these conclusions can be reversed, if the greater price uncertainty of delayed executions or the limited ex ante predictability of alpha/momentum proxy are taken into consideration.
4. ASSUMPTIONS AND IMPLICATIONS OF ITG’S PRICE IMPACT MODEL

While it would be impossible to answer most of the questions about the true price impact without conducting repeated controlled experiments, one can gain valuable insights by modeling own impact along with the impact exerted by other market participants.

Recall the basic thought experiment behind our “alpha” pattern: if we buy (using the VWAP or another “passive” strategy) but consider the possibility of withdrawing from the market (i.e. buying zero shares), what would the sellers who previously traded with us do? Would they avoid trading with any other buyers who are still present in the market? This is possible, but unlikely. To get access to liquidity, the sellers who have been trading with us under the realized scenario would now be forced to act more aggressively, pushing the overall trade imbalance down and, as a result, exerting a downward pressure on the price.

The main stylized facts and assumptions behind our modeling framework required to quantify such a behavior at the aggregate (macro) level are outlined and briefly discussed below.

- A trade of any size within a specified time period exerts an impact on the stock price. Large purchases within a short period are likely to inflate the stock price and result in a large implementation shortfall, whereas the price impact of incrementally small trades executed over a very long time period may result in an unfavorable price drift and sizeable implementation shortfall as well.

- Price changes over a short time period are driven by the trade imbalance, defined as the difference between buyer- and seller-initiated trades, as well as the interaction of trade imbalance with non-directional market conditions (captured by observed volume, volatility, and bid–ask spread). To capture the effect of own trading on the aggregate trade imbalance, we construct a mapping of the realized trade size into the expected trade imbalance, which we call the trader’s trade imbalance “footprint”.

- When we increase the trade size, we start competing more aggressively for liquidity by either submitting more marketable orders, or pricing our limit orders more aggressively. By construction, the changes in other participants’ liquidity taking or liquidity making behavior are reflected in the price impact model.
To capture the effect of an increase in trade imbalance on the execution cost, we model the conditional distributions of mid-quote price movements and the effective spreads for the realized and hypothetical values of trade imbalances obtained from the trade imbalance footprint function.

As we find a strong empirical link between the direction and size of own trades and the concurrent price changes, both limit order executions and dark crosses play an important role as part of any trading strategy, reducing the realized impact of own trades and explaining the concavity of the empirical price impact patterns observed in client execution data.

The price impact exhibits substantial intraday variability. The strongest impact is observed in the morning hours. The impact gradually diminishes over the course of the day, reaching its lowest levels during late afternoon trading.

We model not only the instantaneous price impact, but also the reversion of price trajectories to the realized price path of the stock, after the aftershocks caused by our withdrawal from the market settle down. While doing this, we make a high-level assumption that the market would perceive any shocks associated with our trading path modifications as purely liquidity- as opposed to information-driven.

5. EMPIRICAL EVIDENCE

To evaluate the empirical evidence behind the post-trade price impact model, we used the 20 largest members of the ITG Peer Database [in terms of dollar volume traded] in the first half of 2013. We constructed the single-day orders from the ITG Peer Database and applied the ITG Smart Cost Estimator for Post-trade Analysis (SCE for PTA). Since we had only limited information about the partition between passive and aggressive executions for most of the trades, we assumed a balanced split between limit and market order executions. For all orders, we computed the end-of-day returns [net of the market, sector, and industry common factors] under the realized market conditions. The average values of those returns, and their components attributed to own impact and alpha, are reported for different order size groups in Figure 5.

![Figure 5: Dollar-weighted end-of-day average cumulative returns](image)

Based on execution data by the 20 largest members of the ITG Peer Database between January and June 2013. The returns are net of market, sector and industry effects.

7 We repeated the calculations assuming that high participation rate trades are filled in dark pools. Not surprisingly, the dark pool trading decreased the cumulative return due to own price impact. However, the decrease was moderate (about 10 bps for large orders, on average) and in line with the results reported in other studies on dark pool trading.
The aggregated results indicate that we do not have a bias in the own impact estimates (on average). The red solid line on the plot fluctuates around zero across all order size buckets, suggesting that own price impact returns are consistent with the stock-specific price movement across all order size buckets.

Based on execution data by the 20 largest members of the ITG Peer Database between January and June 2013. The returns are net of market, sector and industry effects.

6. APPLICATIONS AND CONCLUSIONS

In this article, we introduce a new TCA framework that allows estimating one’s own ex post price impact more accurately in aggregate and on individual order level. We showcase an application of this framework to post-trade analysis of a single institutional order execution, and discuss the model’s assumptions as well as the stylized facts captured by the novel approach.

The new post-trade price impact model opens up room for many applications. Here we mention just a few of them, highlighting the breadth and flexibility of the proposed framework:

- Identification of fund manager’s intraday alpha and optimal trade scheduling.
- Case studies of individual orders and outlier analysis.
- Cost attribution for realized and alternative trading strategies, including the implications of delayed trading and opportunity cost analysis.
- Alternative venue analysis: How much do we benefit from trading in the dark?
- Analysis of volume participation strategies: What participation rate is optimal?
- Capacity analysis: How big a footprint can you afford? What are the implications of “scaling up” your own execution strategy due to the client base expansion?

We defer the discussion of these and other applications to our future articles.