
Portfolio Optimization and the Cost of Trading

Abstract

We illustrate the effects of incorporating transaction cost estimates into mean-variance portfolio construction. We begin with an examination of single-period 130/30 portfolios and a two-year monthly rebalancing exercise for a market neutral strategy, both for the Russell 2000 Value universe. Accounting for trading costs *ex ante* delivers superior net returns, broader diversification, lower turnover, and a portfolio robust to noisy alpha signals, relative to standard mean-variance stock selection and portfolio construction. Global portfolios then are analyzed, for three time periods preceding and spanning the current financial crisis. Shifts in regional weights are explainable by trading costs, and differences in portfolio composition are dramatic relative to 'paper portfolios.' These findings imply substantial shifts in investment strategy. Overall, the results confirm that mitigation of transaction costs, leading to improvement in realized returns and better alignment of return with risk, begins at the portfolio construction stage, and, therefore, should not be controlled only at the level of trading desks.

Introduction

Transaction cost research involves the measurement of the cost of executing an order in a security on a post-trade basis, and the estimation of that cost in expected terms for pre-trade analysis, often relating that expectation to some measure of risk.¹ A typical goal of such work is to provide guidance as to how to manage those costs within a trading strategy, including the choice of strategy itself.

Today, 98 percent of large institutions and 88 percent of medium-sized institutions globally are consumers of transaction cost research. Apart from the regulatory mandate of best execution, the wide-spread adoption of this research can be explained by the following simple fact: 40 percent of market participants believe that alpha is lost primarily through trading costs, while 14 percent of respondents attribute the loss of alpha to bad timing of their transactions.² In an environment where 100 basis points difference in alpha can make a hero out of a portfolio manager, an average one-way cost of 25 basis points on a portfolio turned over annually robs 50 percent of that return.

This type of thinking highlights the new challenge of transaction cost analysis. That challenge is to proactively lower transaction costs by helping portfolio managers and traders choose how to construct portfolios and leverage trading strategies. We begin with an old observation: investment performance reflects two factors, the underlying investment strategy of the portfolio manager

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¹See, for example, Domowitz and Krowas (2004), for an early discussion of the extension of the concept of efficient frontier to trading applications, building on the work by Almgren and Chriss (2000).

²"Imperfect Knowledge: International Perspectives on Transaction Cost Analysis," the Tabb Group, March 2008.

and the execution costs incurred in realizing those objectives. In the simplest case, transaction cost analysis is limited to the idea that costs simply eliminate part of the notional or “paper” return to an investment strategy, and therefore, costs should be controlled at the level of the trading desk only. In contrast to this mindset, research on how to generalize and solve the asset-allocation problem in the presence of transaction costs dates all the way back to 1970.³ The focus in most of the published papers has been on technical formulations and the mechanics of problem solving.⁴ Illustration of the link between transaction costs to the fundamental law of active management, as well as to the joint optimization of positions and trades, has been a fairly recent development, however.⁵ Data on the usefulness of transaction cost analysis in portfolio construction also are lacking, because reliable price impact estimates were scarce until recently.⁶ In the absence of stock-specific transaction cost estimates the exercise becomes stylistic, as illustrated in Fabozzi et al. (2006).⁷

The purpose of this paper is to provide empirical evidence on the importance of stock-specific transaction cost estimates in portfolio construction. Consistent with the theory of Engle and Ferstenberg (2007), we incorporate expected transaction costs directly into the portfolio optimization process.⁸ Results from each example are contrasted with those obtained, were one to ignore transaction costs at the portfolio construction stage and simply consider expected net returns. This reference model is a pragmatic view with respect to much of the current state of play: the portfolio manager receives signals with respect to alpha, chooses an optimal set of portfolio weights, takes the desired positions, incurs trading costs, and garners a net return.

Coppejans and Madhavan (2007) demonstrate some theoretical differences between these two approaches, in the context of active management. They show, for example, that even though modest costs significantly degrade performance, improved cost prediction can serve to mitigate the problem. While higher turnover increases breadth, with more active bets per unit time, it leads to greater costs and a reduced ability to capture insights from alpha signals. Our major practical simplification relative to that theoretical work is to condition the problem on expected costs. In other words, we do not vary the uncertainty involved in cost estimates, as a parameter. The variance formulation in Engle and Ferstenberg (2007) includes that of transaction costs, but from the empirical point of view, the addition to the covariance structure has not proved to be meaningful in the context considered here.⁹

We begin with two basic examples. The first is a single-period case, for a 130/30 portfolio, based on the Russell 2000 Value Universe. This simple example illustrates the basic effects that cost modeling may have on the optimization process, including increased diversification and the minimization of overreaction to strong alpha signals, consistent with Coppejans and Madhavan (2007). The second example extends the analysis to a multi-period setting, incorporating a 20-month back-test with monthly rebalancing, for a market and dollar neutral portfolio. Net return performance is shown to

³“An Extension of the Markowitz Portfolio Selection Model to Include Variable Transactions’ Cost, Short Sales, Leverage Policies and Taxes”, Gerry A. Pogue, *Journal of Finance*, 25 (1970), pp.1005-1027.

⁴For instance, in one of the more recent papers Lobo, Fazel and Boyd (2006) study the non-convex mean-variance problem setting which arises in the presence of fixed costs and suggest expressing and solving it as a series of convex optimization problems.

⁵“Is Execution Risk the Same as Investment Risk?”, Robert Engle and Robert Ferstenberg, *Journal of Portfolio Management*, Winter 2007, and “The Value of Transaction Cost Forecasts: Another Source of Alpha,” Mark Coppejans and Ananth Madhavan, *Journal of Investment Management*, 5, no. 1, 2007.

⁶An exception is an early study on global transaction costs, in which it is suggested that proper accounting of transaction costs in the portfolio construction process may lead to sharp changes in the weights given to regional investment objectives in global portfolios; see “Liquidity, Volatility, and Equity Trading Costs Across Countries and Over Time,” Ian Domowitz, Jack Glen, and Ananth Madhavan, *International Finance* 4, 2001.

⁷Fabozzi, Focardi and Kolm (2006), p.74-79, consider a rebalancing example using 18 countries in the MSCI World Index over the period January 1980 to May 2004. For simplicity, round-trip transaction costs are assumed to be constant 50 basis points for all countries and trade sizes.

⁸See for example, Amihud (2002), Spiegel and Wang (2005) and references therein.

⁹This issue is explored in Domowitz, Glen, and Madhavan (2001). Theoretically, the trading cost variance can be decoupled from the portfolio problem if the time allowed for trading is short relative to the holding period of the portfolio; see Engle and Ferstenberg (2007).

be sharply higher in the case in which transaction costs are explicitly incorporated into the optimization problem. Although the position count tends to be higher than in the case where costs are ignored, the dollar turnover falls, reinforcing the single-period lesson with respect to reaction to alpha signals. Simply put, taking costs into account ex-ante increases net returns and diversification, while diminishing the turnover effect on the cost of trading.

We then examine the impact of transaction costs on portfolio composition in a series of global mean-variance portfolio problems. The stock universe shifts from the U.S. market to a global setting, covering North America, Western Europe, Latin America, and East Asia. Results are provided for periods predating and then spanning the credit crisis of 2007-2008. We first adopt the perspective of a U.S. investor, who views the riskless asset as a U.S. Treasury bill, and construct optimal portfolios with and without inclusion of transaction costs in the stock selection process. Trading costs by country are related to shifts in regional investment allocations, in which portfolio weights shift, relative to those calculated without inclusion of transaction costs, by double-digit percentages. These results are strongly affected by the interplay of expected returns and expected costs within and across regions. We therefore turn to an examination of cost-adjusted minimum variance portfolios, which have the advantage of not requiring estimates of expected returns, and provide even sharper results. Moving from a paper portfolio to a cost-aware minimum variance problem results in regional weight changes ranging from a negative 68 percent to over 160 percent.

Two Examples

A. Single-Period Optimization

We consider a single-period enhanced indexing case, consisting of a 130/30 portfolio formed within the Russell 2000 Value Universe. We place no constraints on the problem that would bound turnover or upside position limits, except for risk bounds, in order to illustrate the impact of cost modeling alone.

We perform the optimization at the end of August 2008 and approximate expected stock alphas by calculating the two-year average returns in excess of the risk-free rate preceding the optimization date for each stock and then shrinking the value towards the grand mean across all stocks. The shrinkage intensity is proportional to the standard deviation of the stock's returns. This step helps to alleviate the well-known "error maximization" propensity of portfolio optimizers.

Here, and throughout the paper, expected costs are derived from ITG's Agency Cost Estimator model (ACE).¹⁰ ACE is a dynamic structural econometric model, providing stock-specific expected price impact costs at the level of the order, differing by size of order and market conditions. Permanent and transitory price impacts are explicitly modeled, in such a way as to ensure that the first trade of a multi-trade order affects the prices of all subsequent sub-blocks sent to the market. These expected costs are a function of the trading strategy, set here as a single-day VWAP.¹¹ Model estimates are

¹⁰A complete description of the model, and results comparing expected to actual costs, are contained in ITG ACE---Agency Cost Estimator: Model Description, October, 2007, www.itg.com/news_events/papers/ACE_White_Paper_200705.pdf.

¹¹The qualitative results obtained with 10% volume participation and 40% volume participation strategies are similar to the single-day VWAP strategy. We choose the single-day VWAP strategy to reflect the typical situations faced by investors who track the Russell Index during the rebalancing time. The pressure to maintain a low tracking error to the index forces many portfolio managers to accomplish rebalancing trades in a single day, sometimes pushing up the trading costs relative to the ones obtained with a more serene pace of trading (corresponding, for example, to a 10% volume participation strategy).

available for most names in 42 countries; we exploit the global nature of the model estimates in section III. Forecasted implicit costs from the model are augmented by commissions and fees, to arrive at an expectation of the total cost of trading the order. We assume commission cost to be \$0.02 per share and uniform borrowing costs of 50bp across all stocks.¹²

Covariance estimates are taken from a monthly model within the ITG suite of risk models. Like most such models, market, sector, and industry factors capture differing sources of risk, augmented by growth and size factors on a per-stock basis. The factor covariance matrix is scaled using an option-implied adjustment coefficient to exploit the options market information with respect to the future levels of risk. The loadings are estimated in a time-series framework on a per-stock basis, unlike some cross-sectional paradigms.¹³

The optimization problem maximizes risk-adjusted expected returns and minimizes short borrowing costs, taking the basic form:

$$\max_{\omega} \left(\omega' \mu - \tau \cdot \frac{|\omega - \omega_0|' TC(|\omega - \omega_0|, K)}{100} - \omega_-' BC \right)$$

subject to the constraints

$$\left(\omega' - \omega_B' \right) \Sigma (\omega - \omega_B) \leq \sigma^2,$$

$$\omega = \omega_+ - \omega_-,$$

$$0 \leq \omega_{+,i} \leq \nu, \quad 0 \leq \omega_{-,i} \leq 1 - \nu, \quad \nu \in \{0,1\}, \text{ for } i = 1, \dots, N,$$

$$t' \cdot \omega_+ = 1.3, \quad t' \cdot \omega_- = 0.3 \text{ and } t = [1, 1, \dots, 1]',$$

where

N is the number of stocks in the Russell Value 2000 universe,

$\omega = (\omega_1, \omega_2, \dots, \omega_N)'$ is the vector of portfolio weights,

ω_B is the vector of benchmark weights,

ω_0 is the vector of initial portfolio holdings,

μ is the vector of stock-specific expected returns (in excess of the risk-free rate, in percentage units),

¹²Borrowing costs are hard to estimate since available information is scarce. However, the borrowing cost estimates do not have too much impact on our empirical results. As a robustness check, we use the borrowing costs from D'Avolio (2002), one of the few published papers that discuss borrowing costs. We partition our sample to match the size- and liquidity-groups reported in Table 5 of D'Avolio (2002), assign the corresponding group-average borrowing costs to each stock and run the same optimization with these new estimates. The results obtained are qualitatively similar and are available upon request.

¹³A complete description, including performance testing results, can be found in ITG's Risk Models, Version 3, May 2008, available from the authors.

Σ is the covariance matrix of the stock-specific returns,
 $TC(\cdot)$ is the vector of stock-specific expected transaction costs per share (in bp) which takes into account both implementation shortfall and commission costs and consequently
 K is a function of the traded shares (or equivalently the weight changes relative to the initial portfolio value times the portfolio value),
 BC is the initial portfolio value,
 r_i are the stock-specific borrowing rates (in percent),
 σ is the tracking error bound.

The last three constraints ensure that the total portfolio exposure is indeed 160%. It is achieved by introducing two new portfolio weight variables (ω^+ for positive and ω^- for negative weights) as well as a binary variable ν . We divide the TC component in (1) by a factor of 100 since transaction costs are computed in basis points while returns and borrowing costs are reported in percent.

The initial value of the portfolio is set to \$200 million for the purpose of this example, and we assume that we start from a cash position, i.e. $\omega_{0,i} = 0$ for $i = 1, \dots, N$. We also set $\tau = 1$, which means that both alpha maximization and cost minimization are given equal weight in the objective function. The benchmark weights ω_b are obtained from Russell Investment Company, and the tracking error bound σ is set to be 5 percent.

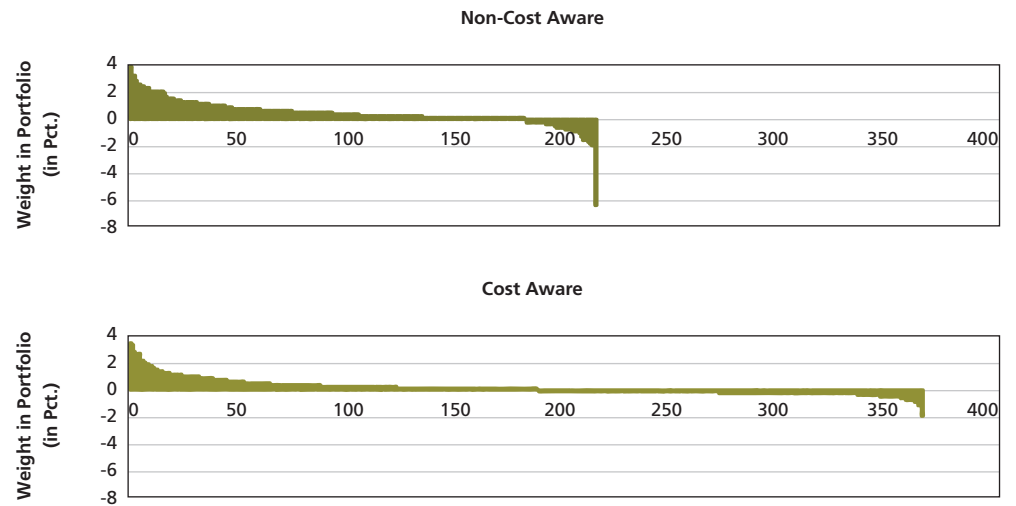
We adopt the following terminology. We refer to the solution of the problem including transaction costs ex ante as being *cost-aware*, while the *non-cost-aware* portfolio is optimized, setting costs to zero and then subtracting expected costs from returns ex post. Virtually by construction, the gross expected return from the cost-aware portfolio is lower than the gross expected return without the inclusion of transaction costs. In our example, the *gross* expected return for the non-cost-aware portfolio is 2.24%, while it is only 1.68% for cost-aware portfolio.

However, the expected cost incurred in acquiring the non-cost-aware portfolio is \$7.4 million, reducing the expected *net* return from a positive 2.24% to a negative 1.46%. In contrast, transaction costs associated with the cost-aware problem total \$2.2 million, keeping the portfolio *net* return positive at 0.57%.

The improvement clearly does not come through a reduction in turnover, since we are considering a single period. However, the cost-aware portfolio incurs costs over a larger set of stocks, illustrated in Figure 1, in which the horizontal axis indexes the names, by number, in the portfolio. The number of positions held in that portfolio is 366, compared to 211 in the non-cost-aware problem.¹⁴ In the absence of cost considerations, expected return is the dominant factor in determining final portfolio weights, subject to the risk constraint. Since costs increase with trade size, at some point the cost to trade will negate the alpha in a name. This type of result is the fundamental driver of fund capacity analysis, but appears here in the form of increased diversification stemming from expected costs of trading. The diversification effect is apparent by looking at the expected total portfolio risk: it goes down from 15.24% in the non-cost-aware portfolio to 13.99% in the cost-aware portfolio. The tracking error relative to benchmark, however, is kept at 5% for both portfolios, as specified in the problem formulation.

¹⁴The actual portfolio securities are not aligned on the chart. For example, stock #20 in the upper chart could be a different security from stock #20 in the bottom chart. Note that 44 securities of the non-cost-aware portfolio have not been selected in the cost-aware portfolio.

Figure 1. Single Period Allocation Comparison



A corollary to the diversification argument is illustrated in Figure 2: the trade sizes relative to ADV are uniformly lower across the entire range of portfolio weights. We define the weights 0-0.3% as small; 0.3%-1% as medium, and the weights exceeding 1% of total portfolio as large.

Figure 2. Trade Sizes (rel.to ADV)

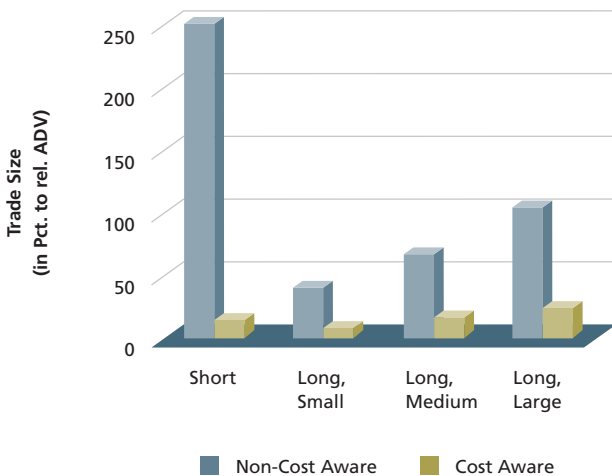
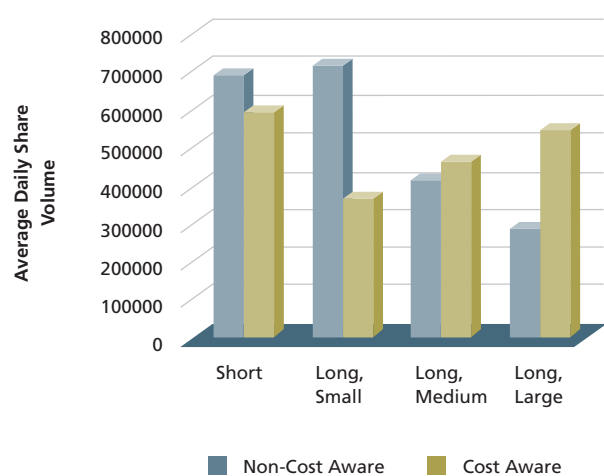


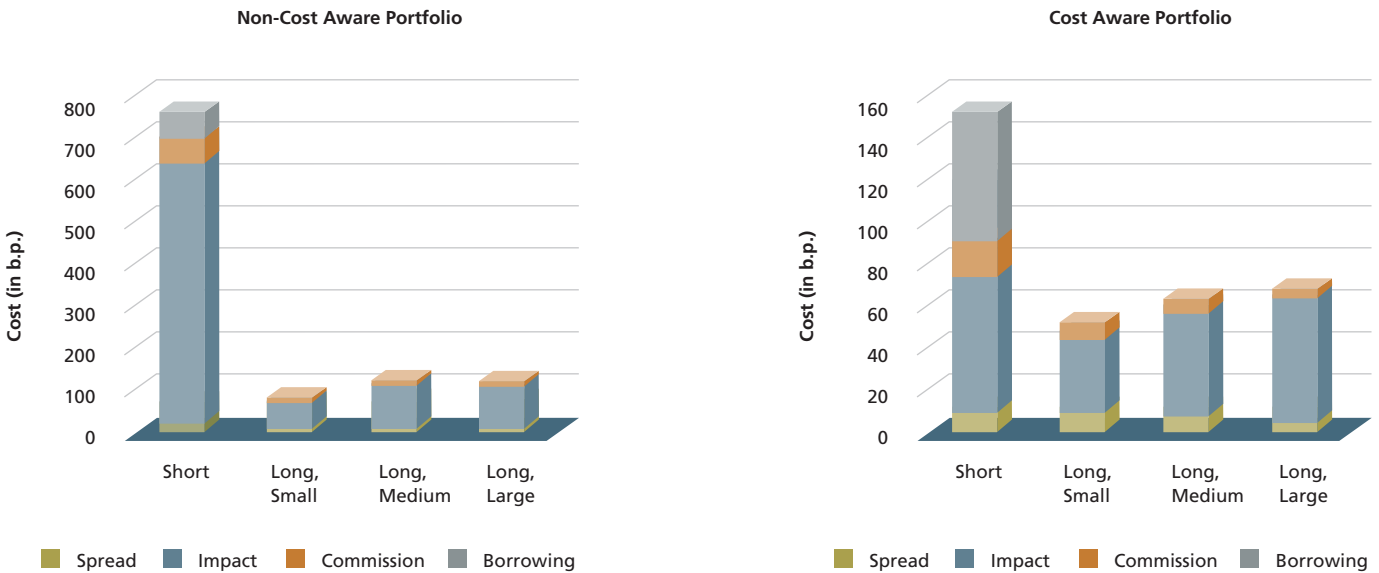
Figure 3. Liquidity of Stocks



When costs are considered, multiple securities with similar alpha characteristics can be given more weight than a single security with a higher alpha estimate. The cost to trade a smaller amount of many securities is less than the cost to trade the same dollar value of a single security, an effect emphasized by typical diversification effects. An alternative interpretation is that overreaction to strong alpha signals is minimized when expected costs are taken into account, consistent with the theoretical prediction of Coppejans and Madhavan (2007).

Figure 3 illustrates how the allocation of securities changes with respect to the stocks' liquidity, proxied here by the 21-day average daily share volume (ADV) numbers. While the non-cost-aware optimizer allocates larger weights (over 1%) to less liquid stocks, the cost-aware optimizer allocates larger weights to more liquid securities. These changes in allocation have direct impact on the size and composition of the transaction cost components, as presented in Figure 4. We decompose transaction costs into four parts: bid-ask spread (which remains largely independent of trade size), commission cost (which we assume is \$0.02 for every security), market impact or implementation shortfall cost (proportional to trade size) and borrowing cost (50bp for every stock). Of course, borrowing costs appear on the short side only.

Figure 4. Cost Decomposition Relative to Portfolio Weights



Besides the obvious effect of reducing the overall level of transaction costs, the key lesson is in the change in the relative weights of the cost components. In particular, the short side of the portfolio benefits the most relative to the non-cost-aware case: the overall level of costs falls from nearly 750bp to 143bp, with the market impact component decreasing from 613bp to 65bp, almost a tenfold drop. This dramatic reduction in costs is, probably, explained by the distribution of expected alphas in our sample. Since the stock market was trending down during the 2007-2008 period, the expected alphas fed into the optimizer are negative for three quarters of all securities in the Russell 2000 Value index. Moreover, the distribution of expected alphas has a relatively heavy left tail (negative alphas) and a relatively light right tail (positive alphas). This results in the non-cost-aware portfolio allocation being concentrated in a few stocks on the short side. Once transaction costs are incorporated in the problem, idiosyncratic risk and stock's liquidity become important, and the optimizer is able to decrease the costs by moving out of extreme weights on the short side. The optimizer simply picks more short-side names, from roughly 30 in the non-cost-aware portfolio to almost 100 in the cost-aware portfolio. For comparison, the number of stocks picked for the long side grows from around 175 to 270, which is a much smaller relative increase. At 100 names picked on the short side of the cost-aware portfolio, the optimizer is nowhere near exhausting the supply of the candidates for short positions; however, 270 names on the long side constitute around 80% of the names with positive expected alpha in the Russell 2000 Value index. Spreading the trading across more names results in a dramatic reduction in the average relative trade size especially on the short side, as illustrated in Figure 2, and transaction costs decline. This shifting of securities and shares in the cost-aware portfolio results in borrowing costs becoming as important as the market impact costs. In addition, the relative importance of average spread and commission costs increases for both short and long positions.

B. Multi-Period Problem

The single period comparison illustrates the basic effects of cost modeling on the portfolio optimization process. We now demonstrate that such desirable results are achievable over time. In this second example, we implement a 20-month back-test, over the period from December 2006 to August 2008, with monthly rebalancing. The period-by-period optimization is mathematically similar to the single-period case, including the use of the Russell 2000 Value Universe. Instead of modeling a 130/30 portfolio with a tracking error constraint, a market neutral / dollar neutral portfolio with an absolute risk bound is used. More precisely, we consider the mathematical problem

$$\max_{\omega} \left(\omega' \mu - \tau \cdot \frac{|\omega - \omega_0|' TC(|\omega - \omega_0|, K)}{100} - \omega' BC \right)$$

subject to the constraints

$$\begin{aligned} \omega' \Sigma \omega &\leq \sigma^2, \\ \beta'_{Market} \cdot \omega &= 0, \\ \omega &= \omega_+ - \omega_-, \\ \iota' \cdot \omega_+ &= 1, \quad \iota' \cdot \omega_- = 1 \text{ and } \iota = [1, 1, \dots, 1]', \end{aligned}$$

where the notation is the same as before and we use the same covariance and transaction cost estimates as in the single-period optimization case.

We begin with a \$50 million budget.¹⁵ We construct a market and dollar neutral portfolio at each monthly step, with an absolute risk upper bound σ of 10 percent and a transaction cost aversion factor of $\tau = 20$, reflecting the fact that we expect to buy and sell the typical security once every two months. The total rebalancing period is 20 months.¹⁶ Expected returns are based on a normally distributed random scheme, here exploiting an information-coefficient formulation,

$$\mu_{i,t} = TC \cdot \mu_{i,t}^{observed} + \sigma_i \cdot \sqrt{1 - IC^2} \cdot \varepsilon_i, \quad i = 1, \dots, N,$$

in which,

IC is the information coefficient,

$\mu_{i,t}^{observed}$ is the realized monthly return observed in month t for stock i ;

σ_i is the monthly volatility of security i , measured over the period considered,

and

$$\varepsilon_i \sim N(0,1).$$

By definition, the correlation of forecasted and realized monthly returns is equal to IC . In what follows, we assume that the information coefficient takes the value of 10 percent (which reflects above average forecasting ability). We draw for each month, and for both the non-cost and the cost-aware case (i.e. $\tau = 0$ and $\tau = 20$), twenty random numbers from the standard normal distribution and obtain twenty draws of $\mu_{i,t}$ using formula (3). The results for both optimization problems are summarized by the means and the 95 percent empirical confidence intervals of net performance, total transaction costs, turnover and number of monthly positions held, demonstrating in all cases statistically significant differences in the outcomes.

¹⁵We choose a different initial cash position and different optimization constraints than in the single-period example in order to illustrate that the inclusion of transaction costs in the portfolio construction process significantly improves net return results for a variety of optimization problems. Results for larger initial cash positions look qualitatively very similar. In fact, the effects are even more pronounced.

¹⁶The turnover results of our cost-aware optimization problem are consistent with this statement. The average turnover is between 150 and 200% for all periods (compared to the maximum possible turnover of 400% per period). See Figure 6 for more details.

The 20-month period chosen for our analysis was quite challenging for the markets. The return on the Russell 2000 Value index, used as a benchmark, from December 2006 to August 2008 was a negative 18.7%. However, due to our high information coefficient, the paper return of our optimal rebalancing strategies remains quite high: 20.9% for the non-cost-aware and 12.8% for the cost-aware multi-period optimization problem. Once the net returns are calculated, the destructive effect of transaction costs on the performance of our rebalancing strategies becomes apparent. To achieve net results, the transaction costs of each rebalance are subtracted from the portfolio value at each rebalancing step during the optimization.

Figure 5. Net Performance Based on Monthly Rebalancing

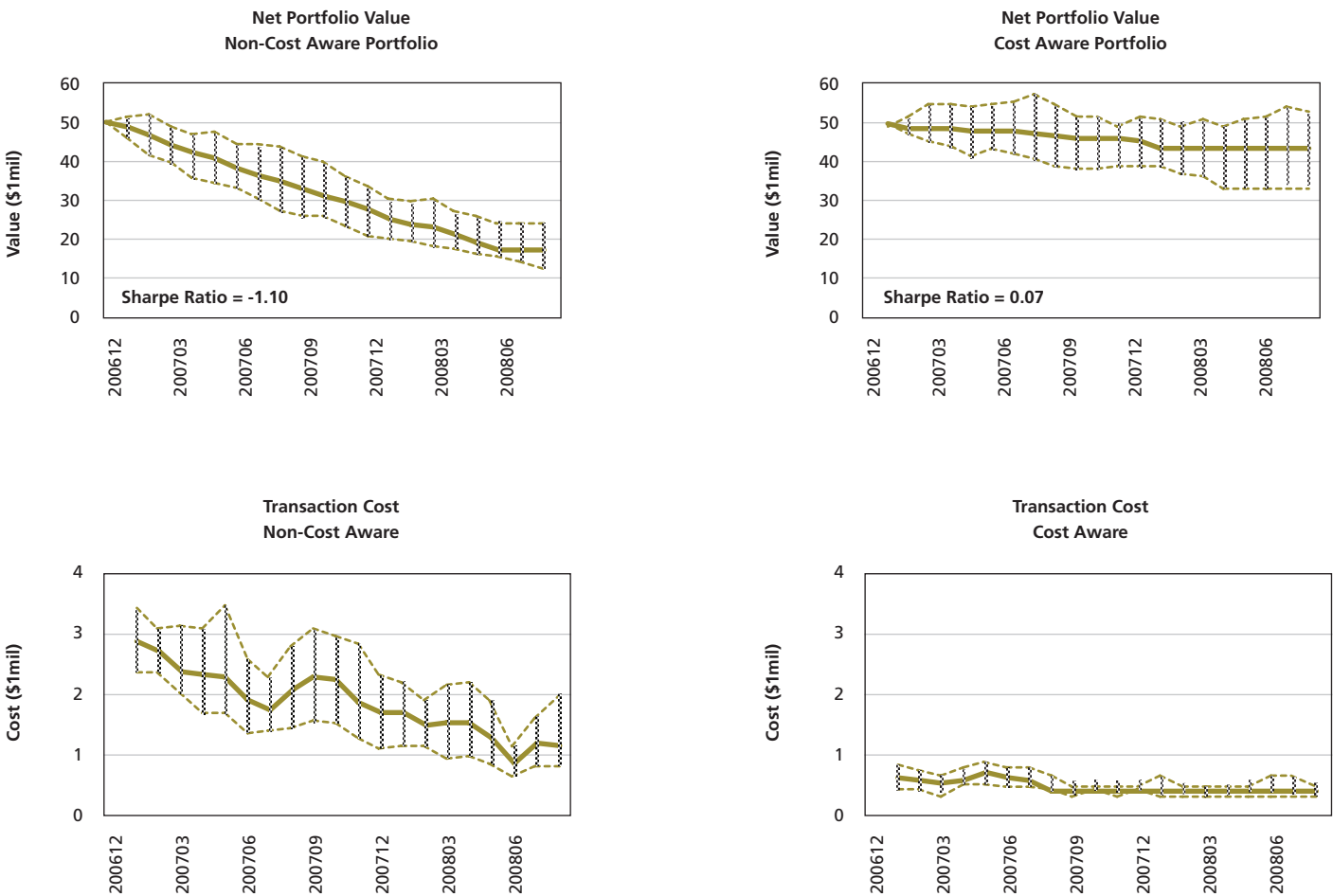
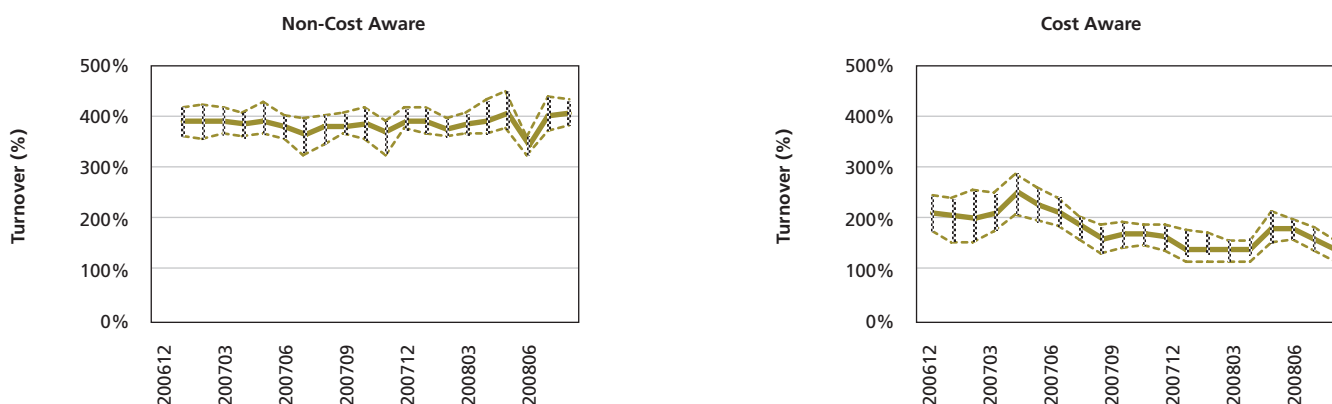


Figure 5 illustrates the relative failure of the non-cost-aware rebalancing strategy in terms of performance. The dotted lines delineate 95 percent confidence bands. The non-cost-aware rebalancing strategy loses on average more than 50% of its initial portfolio value by the end of the out-of-sample period, which translates into a negative Sharpe ratio of about -1.10. In contrast, the cost-aware portfolio performed much better, yielding a modest but positive Sharpe ratio of 0.07.

Figure 5 also helps to identify the sources of the outperformance. First and foremost, the average dollar costs of implementing the cost-aware rebalancing strategy are two to four times lower than the costs of implementing the non-cost-aware strategy. As indicated by the 95% confidence bands, the difference is statistically significant in all 20 months. Second, the costs of implementing the cost-aware rebalancing strategy remain lower despite the fact that its portfolio size starts to exceed the size of the non-cost-aware portfolio by as much as 50% in the last several months of the period considered.

Figure 6. Turnover Comparisons Based on Monthly Rebalancing



The rebalancing exercise emphasizes the lessons concerning the effect of controlling for costs at the portfolio construction stage and introduces a new result pertaining to turnover. Turnover is calculated in percentage terms, as dollars traded relative to the monthly portfolio basis. In a long-only portfolio, this statistic would have a maximum value of 200 percent. That value is doubled for the long-short portfolio with a 2:1 leverage ratio, i.e. the case in which the portfolio is 100 percent long and 100 percent short in dollar terms.¹⁷ Figure 6 demonstrates that the total portfolio turnover goes down from being close to 400% for the non-cost-aware to 250% and even 150% in the later months for the cost-aware rebalancing strategy. In other words, the cost-aware portfolio consistently exhibits significantly lower turnover and the monthly differences can be as large as 270%.

¹⁷In fact, the long-short portfolio turnover can occasionally exceed 400% when, for example, the short side outperforms the long side in the month following the optimization date, thus reducing the net portfolio basis.

Reductions in trading cost did not come from turnover effects in the one-period portfolio, by design. In the case of monthly rebalancing, lower turnover clearly plays a role in this respect. On the other hand, Figure 7 illustrates an increase in diversification through larger numbers of names in the cost-aware portfolio, month by month.

Figure 7. Position Count Progression in Monthly Rebalancing

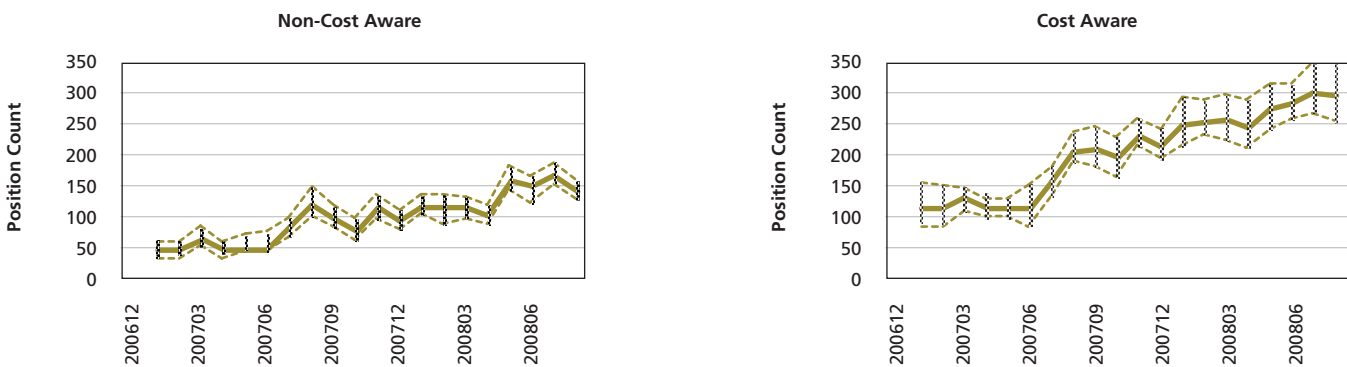
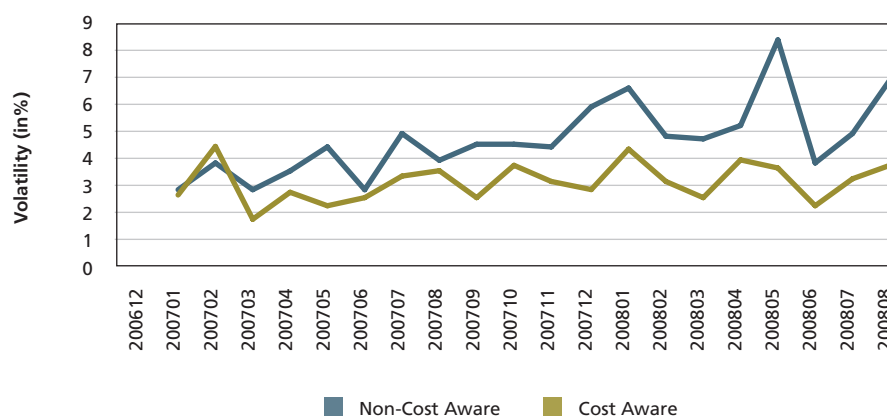


Figure 7 indicates that while the non-cost-aware portfolio contains between 50 and 150 securities, its cost-aware counterpart contains up to 300 securities. During periods of market stress, market conditions overwhelm the transaction cost effect, in the sense that diversification differences across portfolios diminish. For example, the sharp increase in diversification starting in the summer of 2007, as a response to sharply increased risk levels, is not limited to the cost-aware rebalancing strategy. As an important corollary, overreaction to certain alpha signals is lessened when costs are considered at the portfolio construction stage. This also leads to portfolio risk reduction, which we observed (albeit only in-sample) in the previous section. Figure 8 confirms the portfolio risk reduction resulting from higher position counts, decreased turnover and lower sensitivity to extreme alpha signals. The chart shows the monthly realized portfolio volatilities for both the non-cost-aware and cost-aware portfolio computed as the mean absolute portfolio return in a given month. The cost-aware portfolio exhibits lower risk than the non-cost-aware portfolio in all months but one. The result illustrates that traditional portfolio statistics (such as risk) can be significantly influenced not only by the stock selection and hedging decisions, but also by a viable cost-control process.

Figure 8. Monthly Realized Volatility Comparison



Global Mean-Variance Portfolios

One way to gauge the impact of transaction costs is to measure changes in the composition of a portfolio as a result of including costs in the portfolio construction process. We have already seen the potential for such shifts in the context of the single-period Russell 2000 Value example. In this section, the stock universe changes to high-turnover securities in North America, Western Europe, East Asia, and Latin America, aggregated to these regional levels in terms of portfolio composition.¹⁸ For simplicity, we focus on two general solutions to the portfolio problem. The first is the tangency portfolio, adopting the perspective of a U.S. investor, who views the riskless asset as a U.S. Treasury bill. The second is a minimum variance formulation, which has the advantage of not requiring expected return estimates for its construction. We produce results for three different periods: December 2006, December 2007, and August 2008. The first period arguably predates the recent credit crisis, while the second captures some of the initial reaction to that crisis; the last is the most recent data available at the time of this computation, just before the collapse of Lehman.

The rule for selecting securities in each of the six cases studied is based on stock selection criteria for relatively liquid securities in the countries considered. For each country, we rank securities based on the average volume traded during the 21 days prior to each date when the optimization is performed. We then add securities to the universe, starting from the top and going down the list until the total volume traded in selected securities reaches 70 percent of total volume in that market. ETFs and ADRs are excluded in order to keep the analysis as clean as possible.¹⁹ We limit the number of securities from each country to 20, which yields roughly 165 securities in any given sample. Results are computed for a \$500 million portfolio.²⁰

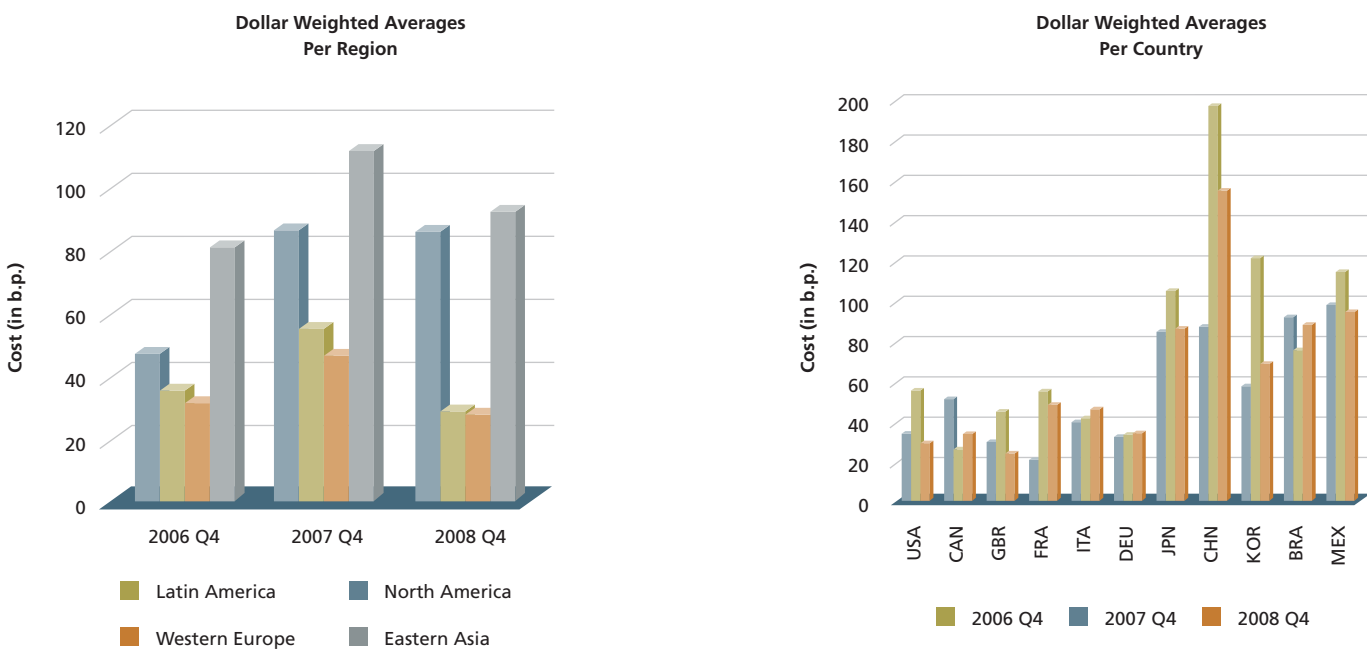
¹⁸More specifically, we use the following country universe: North America includes the US and Canada; East Asia includes Japan, Korea, and China; Western Europe is comprised of the UK, Germany, France, and Italy; and Latin America incorporates Mexico and Brazil.

¹⁹We wanted to ensure that we rely as much as possible on country- or exchange-specific costs to determine the country/region weights in our global portfolios. For instance, excluding ADRs and GDRs will force a U.S. investor to buy/sell Sony in Japan.

²⁰Results are computed for three portfolio sizes, \$50 million, \$100 million, and \$500 million, but the results are qualitatively the same, and even quantitatively similar, so we limit the reporting to the largest portfolio size.

Figure 9 contains the actual dollar-weighted average implementation shortfall costs of the securities for the trimester prior to the optimization dates considered in our analysis. The actual costs are retrieved from ITG’s Peer Group Database.²¹ Average costs are the lowest in Western Europe and the highest in East Asia. In addition, average implementation costs are typically the lowest in 2006 and the highest in 2007. In what follows, we will use these empirical costs to explain some of our optimization results.

Figure 9. Global Transaction Cost Comparison



A. Tangent Portfolios

The tangent portfolio is a solution to the problem,

$$\max_{\omega} \left(\omega' \mu - \lambda \cdot \omega' \Sigma \omega - \tau \cdot \frac{|\omega - \omega_0|' TC(|\omega - \omega_0|, K)}{100} \right)$$

subject to

$$1' \cdot \omega = 1, \quad \iota = [1, 1, \dots, 1]'$$

²¹ITG’s Peer Group Database is ITG’s proprietary client execution database and comprises client execution data from more than 120 different large buy-side institutions.

The notation is the same as in Section II. The tangent portfolio is calculated by rescaling the weights in risky assets obtained from (4) so that they sum to one. The coefficient of risk aversion λ is calibrated so as to ensure that the total risk of the optimal portfolio roughly matches the total risk of MSCI EAFE over the sample period.²² The coefficient of cost aversion, τ , is set to zero, in the case that costs are ignored at the portfolio construction stage, or to successively higher values, which proxy for round-trip turnover levels corresponding to annual ($\tau = 2$), semiannual ($\tau = 4$), and quarterly ($\tau = 8$) rebalancing frequencies. Expected returns are computed similarly to Section I, i.e. via a shrinkage estimator as a rolling window three-year average return in excess of the risk-free rate.²³ Expected implementation shortfall costs are generated from the same source as discussed in the previous sections²⁴, while the necessary covariances are taken from ITG's Global Risk Model and global commission costs have been retrieved from ITG's Peer Group Database.²⁵

In Table 1, we report the weight of securities from each country, aggregated to a regional level, in the tangent portfolio under four cost scenarios: non-cost-aware case, and for the cost-aware cases associated with turnover levels of 100 percent, 200 percent and 400 percent per year. Our discussion is limited to the 100 percent annualized turnover statistics; examination of the table shows that cost effects are sharply exacerbated in the higher turnover cases, but no new qualitative lessons are available there.

Table 1. Regional Weights in the Tangent Portfolio

	December 2006				December 2007				December 2008			
	paper	100%	200%	400%	paper	100%	200%	400%	paper	100%	200%	400%
North America	14.1%	16.0%	16.6%	17.1%	8.6%	8.8%	6.9%	7.7%	39.2%	46.0%	48.7%	54.4%
Latin America	27.0%	24.1%	21.4%	16.4%	5.5%	6.7%	7.4%	6.4%	3.4%	7.2%	9.6%	13.8%
Western Europe	39.4%	43.6%	49.0%	55.0%	49.1%	51.2%	56.5%	59.8%	41.8%	31.0%	26.4%	20.1%
East Asia	19.5%	16.2%	13.0%	11.5%	36.8%	33.3%	29.3%	26.1%	15.6%	15.8%	15.3%	11.7%

Weights on the North American and Western European regions increase by 1.9 percent and 4.2 percent, respectively, once transaction costs are taken into account for 2006. This is a relative increase of 13.5 and 10.7 percent, respectively. The relative decline in East Asian investment is led by China and Japan. To put this result in perspective, transaction costs in Japan are approximately 240 percent higher than in the U.S., for example. East Asia ex Japan exhibits costs that are 67 percent higher

²²A total risk of roughly 13 percent, for example, translates into $\lambda = 2$.

²³We wanted to ensure that we rely as much as possible on country- or exchange-specific costs to determine the country/region weights in our global portfolios. For instance, excluding ADRs and GDRs will force a U.S. investor to buy/sell Sony in Japan.

²⁴Results are computed for three portfolio sizes, \$50 million, \$100 million, and \$500 million, but the results are qualitatively the same, and even quantitatively similar, so we limit the reporting to the largest portfolio size.

²⁵ITG's Peer Group Database is ITG's proprietary client execution database and comprises client execution data from more than 120 different large buy-side institutions.

than in North America over the period. In contrast, Western Europe’s trading costs are running at about 31 basis points during this period, which is a bit lower than in the U.S. This accounts for the increase in Europe’s portfolio weight relative to other regions.

U.S. trading costs rose in the fourth quarter of 2007, relative to 2006, reflecting increased market friction due to market volatility that began in August of 2007. Western Europe’s costs also rose, but any portfolio effects due to that increase were offset by a jump in Asian costs by 38 percent. Korea’s weight in the portfolio dropped by 28 percent, for example. This led to a 9.5 decrease in the Asian weight, relative to the paper portfolio, while investment in North America and Europe grew by small amounts, once costs were factored into the calculation.

In August 2008, average costs in Western Europe ex UK remained at the same high levels as in 2007, with transaction costs being 80 percent higher than in December 2006 (mostly attributed to France and Italy). However, expected returns fell for that region in 2008, leading to a relatively low weight in the paper portfolio, further reduced by 26 percent once costs were taken into account. In contrast, U.S. costs fell by about 13 percent. In the face of a sharply lower weight for Europe, and a decrease in costs for U.S. and Canadian securities, the North American weight rose over 17 percent relative to the paper portfolio. Interestingly, the Asian weight remained basically unchanged, due to country-specific effects in the region. In particular, Japan’s costs remained approximately the same as in December 2006, increasing its relative share in the portfolio, while Korea’s and especially China’s costs increased relative to December 2006.

On a global basis, the story is basically a decrease of liquidity in Western Europe ex UK, which has the effect of sharply lowering European investment exposure, relative to a paper portfolio and other regions. Of course, this type of result is strongly affected by the interplay between costs and expected returns within and across regions. For example, the results for Western Europe are influenced by a decline in expected returns by approximately half in August 2008, relative to the previous eight months. We now turn to a set of scenarios in which the effect of costs is examined without consideration of return expectations.

B. Cost-Adjusted Minimum Variance Portfolios

We define the cost-aware minimum variance portfolio as the solution to the problem,

$$\min_{\omega} \left(\omega' \Sigma \omega + \tau \cdot \frac{|\omega - \omega_0|' TC(|\omega - \omega_0|, K)}{100} \right)$$

subject to

$$t' \cdot \omega = 1, \quad t = [1, 1, \dots, 1]'$$

in which we use the same notation as before. The formulation (5) is a special case of (4) where we have a strong prior that $\mu = 0$ (i.e., the expected returns for all securities are set to be zero). The model has the advantage of not requiring estimates of expected returns, unlike all previous examples in which the effects of alpha and transaction costs compete with each other. The optimization problem is solved with $\tau = 0$ and $\tau = 2$, representing the paper or non-cost-aware portfolio and a cost-aware portfolio with annual turnover.²⁶ Results are presented in Table 2 for a \$500 million budget.

Table 2. Regional Weights in the Minimum Variance Portfolio

	December 2006		December 2007		December 2008	
	paper portfolio	100% turnover	paper portfolio	100% turnover	paper portfolio	100% turnover
North America	51.3%	53.4%	42.7%	48.6%	57.6%	64.3%
Latin America	6.3%	2.0%	2.5%	1.4%	2.7%	4.9%
Western Europe	12.1%	31.9%	21.4%	35.9%	10.0%	14.2%
East Asia	30.3%	12.7%	33.4%	14.1%	29.7%	16.6%

Overall portfolio risk increases as one moves from the paper portfolio to the cost-aware formulation, by construction. The increase is 22 percent for 2006 and 2007, falling to 16 percent in 2008. Risk is growing, year over year, but this is true regardless of transaction costs, given changes in the global economic environment.

The cost-adjusted minimum variance formulation drives the results in a stark fashion, which was partially masked by the previous inclusion of expected returns. China is a good example in this context. Costs are sufficiently great so as to reduce China's weight in the portfolios by an average 51 percent over the three periods. These adjustments are not from a similar base, however, in terms of the paper portfolio. China's weight, without regard for transaction costs, decreases by 48 percent in 2007, relative to 2006, and then by over 80 percent in 2008, relative to 2007. Simply put, China loses its diversification benefit over this period, and its high cost of trading exacerbates this effect. The correlation between the Shanghai SSE composite and indices such as the SPY and EAFE increases sharply between the end of 2006 and August of 2008.²⁷ Once one factors in the high trading costs in Japan and Korea noted earlier, East Asia weights in the cost-aware portfolios fall sharply, by between 44 and 58 percent, depending on the year.

The East Asian effect on the minimum variance portfolio is strong enough to reverse some results relative to the optimal tangent portfolio. Western Europe's cost-aware weight increases in 2008, for example, relative to the paper portfolio, by 42 percent. Although changes in the North American

²⁶A higher turnover assumption is typically not necessary to maintain a minimum-variance portfolio, and provides little insight in this case.

weights are small in percentage terms, moving from a paper formulation to a cost-aware minimum variance problem results in substantial weight changes ranging from -68 percent for Latin America in 2006 to 168 percent for Western Europe in 2007.

Conclusion

Recent theoretical results have advanced our understanding of the role of transaction costs in portfolio optimization. A simple perspective of reduced returns has moved to an appreciation of the implications of cost-induced shifts in the efficient frontier, in the context of portfolio rebalancing and the fundamental law of active management. The contribution of this paper is to identify new implications in some cases, and empirically illustrate existing theoretical ones in others.

Examples of single-period 130/30 portfolios and multi-period market neutral long-short problems deliver stark results. Compared to problems which ignore trading costs, formulations which incorporate them explicitly in the optimization process increase net returns, broaden diversification, and create more stable portfolios in the sense of lower turnover and mitigation of noisy alpha signals.

In the context of global portfolios, shifts in regional weights are explainable by costs across three time periods preceding and spanning the credit crisis. Differences in composition, once transaction costs are taken into account, are pronounced with and without the countervailing effects of expected returns. Changes in portfolio weights are shown to be dramatic. Declines in individual weights ~~decline~~ by as much as 68 percent in some cases, and can increase by 168 percent in others, depending on region and time period. These changes imply substantial shifts in investment strategy.

Originally driven by regulatory concerns, the interest in transaction costs and their importance for institutional returns has been renewed with the advent of the current financial crisis and the corresponding spike in equity market volatility. There is much interesting work to be done in the future. Proper consideration of transaction costs can be used in establishing the viability of certain investment strategies. One example is fund capacity analysis and the optimal timing of fund closures. Another possibility is illustrated by Hou and Moskowitz (2005), with respect to the construction of portfolio strategies that exploit the relationship between the cost of trading a stock and its expected returns.

We offer a basic and very practical message. Cost reduction is not only the responsibility of trading desks. Mitigation of frictional costs, leading to improvement in realized returns and better alignment of return with risk, begins with the inclusion of transaction costs in the stock selection and portfolio construction process.

²⁷The correlation with SPY is virtually zero at the beginning of the period, rising to over 40 percent by the end, for example.

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