Liquidity Commonality and Return Co-movement*

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Abstract

We show that liquidity commonality is due to co-movements in supply and demand induced by cross-sectional correlation in order types (market and limit orders), while return commonality is caused by correlation in order flows (order direction and size). Since return and liquidity commonality are caused by different economic forces, it is possible for assets to have little return correlations but high liquidity commonality. Based on extensive simulations and empirical evidence using ASX data, we conclude that it is important to consider not only liquidity, which has been shown to be a priced factor, but also liquidity commonality in asset pricing applications.

1 Introduction

While portfolios feature prominently in much of the finance and particularly the asset pricing literature, microstructure research has focused almost entirely on the study of single security settings. Until recently, little attention had been paid to the basic interactions among individual securities' liquidity, or liquidity commonality. As part of the existing body of work concerned with liquidity commonality, Chordia, Roll and Subrahmanyam (2000) assume a special role of the market *a priori* and find that individual liquidity co-moves with market liquidity, and that liquidity commonality remains significant even after controlling for individual liquidity determinants. In contrast, Hasbrouck and Seppi (2001) do not assign an *a priori* role to any one factor. Instead, they conduct a principal component analysis and find empirically that the liquidity of the Dow 30 stocks exhibits a single common factor, though the commonality is not very strong. Huberman and Halka (2001) also find that liquidity across stocks has a systematic component in a sample of daily NYSE data. Regarding the causes, Coughenour and Saad (2004) argue from a liquidity supply perspective that common market makers are one reason for liquidity commonality. As for markets without any designated liquidity suppliers, Brockman and Chung (2002) and Bauer

(2004) document the existence of liquidity commonality also in the purely order-driven settings of the Swiss Stock Exchange and the Hong Kong Stock Exchange, respectively.

In this paper, we go beyond merely providing evidence for the existence of liquidity commonality, but examine its cause and evaluate its implications for asset pricing by demonstrating a linkage between liquidity commonality and return co-movement, a key determinant of portfolio choice. Specifically, we begin by formulating a measure of liquidity defined as a function of demand and supply schedules. We apply the measure in two settings where such schedules can be observed: a simulated limit order book where we have control over order type (market or limit orders) and order flow (order direction and size), and a comprehensive dataset of limit and market orders from the Australian Stock Exchange. The simulation results show that order type co-movement leads to liquidity commonality but not return commonality, while order flow co-movement leads to return commonality but not liquidity commonality. In particular, when returns are negatively correlated and thus the assets desirable for diversifcation purposes, liquidity commonality can be especially high.

In the empirical part, we first show that order type and order flow are not significantly correlated with each other, but that they are really different aspects of a given order in both an economic and a statistical sense. We can easily find examples of stock pairs that have small return correlation but exhibit significantly positive liquidity correlation. Strikingly, these stocks' order-flow correlations are close to their return correlations but not their liquidity correlations, and their order-type correlations are close to their liquidity correlations but not their return correlations.

Next, we regress return (liquidity) correlations on order-type correlations and order-flow correlations. The results show that order-flow correlations dominate order-type correlations in explaining return correlations, and order-type correlations dominate order-flow correlations in explaining liquidity correlations. Stocks that do not correlate in returns can exhibit liquidity co-movement because return co-movement and liquidity co-movement are caused by different economic forces. Hence, liquidity co-movement does indeed pose a problem for traditional diversification strategies based solely on return interactions. This conclusion is the major contribution of the paper.

Finally, we examine if there is any asymmetry in liquidity commonality. We find that liquidity commonality reveals itself more through deterioration than improvement across stocks. This asymmetry stands out as an important result because it highlights the downward risk of liquidity commonality. Liquidity commonality would be more welcome if it manifested itself by simultaneous increases across stocks, unfortunately, the opposite is the case.

The remainder of the paper is organized as follows. Section 2 introduces our measure of liquidity based on demand and supply primitives. In Section 3 we provide a theoretical argument that order type co-movement leads to liquidity commonality and order flow co-movement leads to return commonality. Sections 4 and 5 detail the simulation-based evidence and the empirical results from the ASX, respectively. Section 6 concludes.

2 A Measure of Liquidity and Its Determinants

Before discussing commonality in liquidity, we first need a measure of liquidity. Quoted spread, depth, and ratios based on both spread and depths are standard liquidity measures applied in dealer markets, but they are problematic in that they do not reflect the liquidity supply beyond the inside quotes, and they do not incorporate the demand for liquidity. In addition, many trades, especially large ones, are not executed at the quotes. Further, in a dealer market, only the best quotes are typically observable which restricts the measurement of liquidity beyond the quotes. The effective spread improves on the quoted spread as a liquidity measure in that it is not restricted to the inside quotes and at least partially reflects liquidity demand by accounting for the price impact of trades. However, the effective spread is only an *ex post* measure of liquidity for a particular transaction size, not an *ex ante* estimate of liquidity for *any potential* trade size.

We intend to measure liquidity from a basic economic supply and demand perspective. Since supply and demand are expressed most visibly in a limit-order market, we use that market structure for illustration purposes, with the understanding that, theoretically, the measure applies to other markets as well, since obviously supply and demand exist in every market. In a pure limit-order market setting, Irvine, Benston and Kandel (2000, hereafter IBK) and Coppejans, Domowitz and Madhavan (2000, hereafter CDM) propose a size-related, *ex ante* liquidity measure that aggregates all limit orders on the book. We basically follow IBK and CDM's ideas. Figure 1 shows an example of the instantaneous demand and supply schedules for a security in a limit order market. The demand (supply) schedule is expressed as a monotonically decreasing (increasing) step function.

Let A_i (B_i) denote the i^{th} best offer (bid) price, and S_i (D_i) denote the corresponding limit order quantities. Intuitively, liquidity in this market is related to the area between the two schedules. The farther away supply is from demand for a given quantity, the lower is liquidity. As supply and demand schedules approach each other, liquidity becomes infinitely high. We can also view the area between the supply and the demand schedules as the concession an "impatient" investor has to make to get her order executed quickly against the orders queueing on the book. This concession also measures liquidity, and a lower concession implies higher liquidity for the asset.

Formally, using the notation introduced above, and given the market supply and demand situation, if an investor simultaneously buys and sells q shares of the stock, he has to pay

$$\left[\sum_{j=1}^{k-1} S_j A_j + (q - \sum_{j=1}^{k-1} S_j) A_k - q\frac{A_1 + B_1}{2}\right] + \left[q\frac{A_1 + B_1}{2} - \left(\sum_{j=1}^{k'-1} D_j B_j + (q - \sum_{j=1}^{k'-1} D_j) B_{k'}\right)\right]$$
$$= \left[\sum_{j=1}^{k-1} S_j A_j + (q - \sum_{j=1}^{k-1} S_j) A_k\right] - \left[\sum_{j=1}^{k'-1} D_j B_j + (q - \sum_{j=1}^{k'-1} D_j) B_{k'}\right], \tag{1}$$

where k and k' denote the indices of the last sell and buy limit orders the round-trip transaction executes against, and $(A_1 + B_1)/2$ is the prevailing mid inside quote. Geometrically, (1) represents the area between the supply and demand functions up to trade size q (the shaded area in Figure 1). Writing (1) in integral form, we have

$$l_t(q) = \int_0^q \left[S_t(Q) - D_t(Q)\right] dQ. \tag{2}$$

Equation (2) measures the liquidity at time t for a hypothetical round-trip trade of size q.[1] One-

[1] While (2) is actually a measure of the illiquidity of an asset in that higher values of l imply a lower liquidity,

way liquidities can be represented in a similar fashion. The sell-side liquidity and the buy-side liquidity are respectively the upper-half and the lower-half of the shaded area separated by the straight line $P_{0,t}(q) = (A_{1,t} + B_{1,t})/2$, so that

$$l_t^{\text{sell}}(q) = \int_0^q [S_t(Q) - P_{0,t}]\, dQ, \tag{3}$$

$$l_t^{\text{buy}}(q) = \int_0^q [P_{0,t} - D_t(Q)]\, dQ, \tag{4}$$

with

$$l_t(q) = l_t^{\text{sell}}(q) + l_t^{\text{buy}}(q). \tag{5}$$

From (3), (4), and (5), we can see that the liquidity measure is size-related *ex ante* and that it goes beyond the inside quotes. It improves upon the quoted spread because it aggregates all the orders on the supply and demand sides, and thus provides a more complete measure of liquidity It improves on the effective spread because it is *ex ante* for *any* hypothetical order size q, while the effective spread is *ex post* for some *realized* trade size that occurred.

If there are no limit orders on either side of the book, the security is infinitely illiquid; if only one side of the book is empty, for example the supply side, then $l^{\text{sell}} = 0$ since there are no sell orders to match with immediate buys. However there is still buy-side liquidity. In this case, our liquidity measure can be adjusted to, for example,

$$\int_0^q [B_{1,t} - D_t(Q)]\, dQ.$$

Note also how the liquidity measure differs from the slope of a supply (demand) curve. To obtain the slope, we would need to extrapolate the supply/demand step functions first. However, depending on different methods of extrapolation, we may obtain different slopes. Besides, extrapolation requires additional assumptions regarding the form of the smoothing function. We feel it is better

we will refer to l as a liquidity measure.

not to make any assumption but rather use exactly what is actually observable. Therefore we use the area instead of the slope.

Figure 1 about here

Given the current supply and demand schedules, an incoming order will, ceteris paribus, shift the supply or demand curve, resulting in a change in the area between the two curves and thus a change in liquidity. Consider the examples shown in Figure 2. When a market buy order arrives, it will consume liquidity on the supply side. As a result, the supply curve shifts upwards, the area between the two curves becomes larger, and the security becomes more illiquid. A (non-marketable) limit buy order, on the other hand, will add liquidity to the demand curve. If it is placed within the inside spread, the entire demand curve will shift upwards and liquidity will increase. If it is submitted behind the best price, the left-most part of the demand curve is unaffected, but the remainder of the curve shifts upwards. This implies that liquidity for small trade sizes is unchanged, while liquidity for larger trade sizes increases. We can see that even though both orders are buys, they have different effects on liquidity due to their different types.

Figure 2 about here

What directly and solely determines whether liquidity changes or not is order *type*, i.e., whether an order is a market order (consuming liquidity) or a limit order (supplying liquidity), not order direction. In other words, if we only know order type, we will know the change to liquidity, but if we only know order direction, we cannot tell how this order will affect liquidity. Order size, on the other hand, measures the *magnitude* of the change in liquidity, but even without knowing order size, we can still infer whether liquidity increases or decreases based on order type.

3 Commonality in Liquidity

3.1 Order type co-movement matters most for liquidity commonality

Based on the discussion in the previous section, it should become clear that commonality in order flows is not as crucial as commonality in order types in causing liquidity co-movement. For simplicity, assume order size is fixed. Order flow co-movement refers to the correlation between the arrival of buy and sell orders in individual securities. But order flow co-movement does not imply liquidity co-movement, because buys and sells can take the form of both market and limit orders. On the other hand, order type co-movement among securities refers to the correlation of market and limit order arrivals. Now, even without knowing order directions or order sizes, we know that liquidity in the securities will tend to be correlated.

For example, for two securities i and j, the covariance between l_i and l_j can be written as[2]

$$Cov(l_i, l_j) \approx Cov(S_i, S_j) + Cov(D_i, D_j) - Cov(S_i, D_j) - Cov(D_i, S_j). \tag{6}$$

To see how the signs in front of each term in (6) are determined, consider the following. If both supply curves shift upwards (downwards), both securities' liquidity is reduced (increased). If one supply curve shifts upwards while the other shifts downwards, liquidity in the first stock is reduced and liquidity in the second stock is increased. Thus, the covariance between the two supply curves is positively related to the covariance in the two securities' liquidity. Similar considerations for the remaining cases explain the remaining signs in (6).

There is some empirical and theoretical work which has shown that orders of the same type tend to arrive together across stocks. Biais, Hillion & Spatt (1995), Foucault (1999), and Ranaldo (2003) have shown that in a limit order book market, traders offer liquidity by submitting limit orders when liquidity is scarce and consume liquidity by submitting market orders when liquidity is plentiful. Extending these results to two stocks A and B, we conjecture that if A and B's liquidity

[2] Appendix A lays out some theoretical work to derive liquidity commonality.

is found to increase or decrease at the same time, it should be due to traders simultaneously submitting orders of the same type for A and B. This order placement behavior will cause A and B's liquidity to co-move, which, in turn, will induce further order-type co-movement through traders' order placement behavior. It is a self-enforcing process and this endogeneity of liquidity and order submitting strategy suggests a natural link between the two in explaining liquidity commonality. However, in this paper, we do not examine what causes the order placement strategy to co-move, nor do we intend to examine the causality between cross-stock order placement strategy and liquidity. We leave this question for future research.[3]

3.2 Order flow matters most in return co-movements

Returns are defined as percentage changes in prices, and prices are determined by supply and demand. Order direction – buy or sell – indicates whether the order is supply or demand, and order size indicates the size of the supply and demand. Thus, order flow affects the shape of the supply and demand curves, their intersections, i.e., the equilibrium prices, and price changes, i.e., returns. That order flow co-movement drives return co-movement is also an empirical result of Hasbrouck and Seppi (2001). Since we are in a limit-order market, we calculate returns as the percentage changes in mid-inside quotes. In this case, order type may impact returns as well because market orders, as well as aggressive limit orders, tend to move the inside quotes. Nevertheless, we conjecture that order type is of secondary importance relative to order flows in determining prices and returns, and this is supported by our simulations and the empirical evidence.

In the next section, we will simulate a limit-order book for two stocks and show that order type co-movement leads to liquidity co-movement and that order flow co-movement leads to return co-movement. Since return co-movement and liquidity co-movement are caused by different forces, it is possible that returns are either negatively correlated or uncorrelated, even when liquidity is positively correlated, and vice versa. This result calls for an adjustment in the traditional

[3]We think endogeneity is an explanation for liquidity commonality rather than a problem.

diversification strategy.

4 Simulation of the Limit Order Book For Two Stocks

We simulate the evolution of the limit order books for two hypothetical stocks, A and B. Before each simulation run, we initialize both sides of the two books with a large number of limit orders, essentially to ensure that neither side of the book becomes empty during the simulation. For concreteness, we choose an initial mid-price of \$20, and space these initial orders (assumed to be for 1000 shares each) a tick size of \$0.10 apart. At each iteration $t = 1, \ldots, 500$ during a simulation run, a new order arrives in each stock. Orders are characterized by three parameters: order type (market or limit), order direction (buy or sell), and order size, which we assume to be uniformly distributed between 100 and 3000 shares.

For limit orders, we distinguish two cases: If the quantity demanded (offered) at the current best bid (ask) is more than 3000 shares or the current spread is greater than or equal to 2 cents, the new limit order to buy (sell) is assumed to be submitted at the current mid-price. Otherwise, we assume that the new limit order is submitted at the current best bid or offer price. These assumptions are consistent with empirical evidence on limit order trader submission behavior. For example, Biais, Hillion, and Spatt (1995) find that traders become aggressive in order submission when the current inside spread is large or the current own-side queue is too long.Market orders are simply matched against orders waiting on the book until they are completely filled.

At each time t during a run and for each stock, we calculate the mid-price return and liquidity as defined in Equation (2) for 10 hypothetical trade sizes $q = 1, 301, \ldots, 2701$ based on the current state of the book. Using these 500 time series observations, we compute Spearman correlation coefficients

$$\rho^{l_q} = \mathrm{Corr}_t(l_{A,q,t}, l_{B,q,t}), \qquad \text{and} \qquad \rho^r = \mathrm{Corr}_t(r_{A,t}, r_{B,t}).$$

We perform three different sets of simulations, each consisting of 500 simulation runs, and report the average Spearman correlations over the 500 runs.

4.1 The three sets of simulations

In the first set of simulations, we randomize order flows, but control order types. Specifically, let p_1 denote the probability that the two orders, i.e., the order submitted in stock A and the order submitted in stock B, are of opposite type. We vary p_1 between 0.1 and 0.9. Each order is equally likely to be a buy or a sell.

In the second set of simulations, we randomize order types, but control order flows. Let p_2 denote the probability that one order is a buy order while the other is a sell order, and $1 - p_2$ the probability that both orders are either buys or sells, where either case is equally likely. Order size is restricted to be identical across stocks. We leave order types purely random with half of them being the same (two market orders or two limit orders) and the other half being different for A and B (one limit and one market order). As in the first set, we vary p_2 between 0.1 and 0.9.

In the third set of simulations, we control both order flows and order types. Let p_3 denote the probability that one order is a market order while the other is a limit order with both orders having the same direction, and let $1 - p_3$ denote the probability that both orders are of the same type and opposite direction. As in the second set of simulations, orders for A and B have the same size.

4.2 Results

Table I reports the result from the first set of the simulations. There is a clear monotonic relationship between p_1, the probability that the two orders are of opposite types, and the Spearman correlation coefficient for liquidity. The latter is at its maximum (0.4558 for $q = 1$) when $p_1 = 0.1$, close to zero when $p_1 = 0.5$ and at its minimum (-0.3601 for $q = 1$) when $p_1 = 0.9$. This relationship holds for all hypothetical trade sizes. In contrast, return correlations are close to zero (all smaller than 0.01 in magnitude) for all values of p_1. The results from the first set of simulations are consistent with our conjecture that order-type commonality explains liquidity commonality but not return commonality. In the next set of simulations, we are going to show that order-flow commonality cannot explain liquidity commonality, but can explain return commonality.

Table I about here

Table II shows our results from the second set of the simulations. The liquidity correlations are mostly negative and are all negligible in magnitude regardless of the value of p_2, which denotes the probability that one order is a buy and the other is a sell. The liquidity correlations reach the highest absolute values when $p_2 = 0.4$, but even then they are only around -0.05. In contrast to the liquidity correlations, we see a monotone decreasing trend in return correlations, from 0.2144 when 90% of the order flows are of the same direction to almost zero when 50% of the orders are of the same direction, finally reaching -0.2106 when 90% of the orders are of opposite direction. The monotone decreasing trend is consistent with our conjecture that order-flow co-movement causes return co-movement. The negligible magnitude of liquidity correlations shows that order-flow co-movement does not lead to liquidity co-movement.

Table II about here

Table III reports the results from the third set of simulations. The decreasing trend in liquidity correlations and the increasing trend in return correlations are similar to those observed in Simulation I and II. In addition to these similar trends, the liquidity correlations in Simulation III are all of a greater magnitude than those in Simulation I. This stems from the secondary effect of order flow on liquidity because in Simulation III, order size, which determine the magnitude of liquidity changes, are the same for both securities. The larger magnitude in return correlations for Simulation III with $p_3 < 0.5$ than for Simulation II with $p_2 > 0.5$ are consistent with the secondary effect of order type on return, because in the former case more than 50% of the order flows are of the same type while in the latter case they are always equally divided.

It is worth highlighting one particular result in Table III. From the row with $p_3 = 0.1$ we see that when returns are negatively correlated, as ideal for classical mean-variance diversification purposes, liquidity correlation is highly positive, and this liquidity commonality will cause difficulty in realizing the diversification benefits: it is simply not easy to trade such a portfolio of stocks.

Table III about here

5 Empirical Evidence From the Australian Stock Exchange (ASX)

5.1 Data

We now turn to an empirical analysis of Australian Stock Exchange (ASX) data. ASX was formed in 1987 as a result of the amalgamation of six independent stock exchanges that formerly operated in Australia's six state capitals, and by the end of December 2000, had a domestic market capitalization of $670.9 billion, and nearly 60,000 trades every day. ASX operates as a pure limit order market. Its computer-based order submission and matching system is called SEATS. Every ASX member has equal access to the SEATS screen, which shows all the concurrent orders for ASX-listed stocks. SEATS opens every day at 10:00 am and the opening session lasts for about 10 minutes. Securities open in five groups, according to the starting letter of their ASX ticker symbols. From opening until 4:00pm is the normal trading period in which trades take place when orders are automatically matched with each other. The vast majority of trades take place during the normal trading period. From 4:00pm to 4:05 pm, SEATS prepares to close and the trading stops. The closing single price auction takes place between 4:05pm and 4:06pm. Orders cannot be matched automatically outside of the normal trading period, but trades may occur if brokers contact each other directly. The data used in this paper is for the 19 stocks that consistently make up the ASX-20 index from March 2000 to December 2000, 211 trading days altogether. One stock is dropped from the sample because it was replaced as an index constituent during the period.

5.2 Summary Statistics

We reconstruct the limit order books from the SEATS Order Book Entry (OBE) files which record every order's size, limit price, as well as starting and ending time with a resolution of 0.001th of a second. The data allows us to infer the number and characteristics of orders in the book at any specific time.[4] We only use the normal trading period on each trading day, and take six snapshots of the limit order book of each stock at hourly intervals from 11:00am to 4:00pm. For simplicity, we truncate the books to at most ten price steps. From the snapshots of the truncated limit order books, we calculate liquidity based on Equation (2) for 11 hypothetical sizes from $q = 1, 2001, \ldots, 20001$ shares. The selection of the 11 sizes is justified when we look at the average order size distribution and the trade size distribution in Table IV Panel A. The average median order size is 2111 shares and 10% of orders are larger than 17838 shares. Trade sizes are smaller than order sizes on average because an order can be matched with several opposite orders which will result in several trade records, but the 90th percentile of trade size is still 10000 shares. The truncation to ten steps is for simplicity and it turns out that more than 90% of the time, the order book up to step ten has more than 20000 shares, which is big enough for our largest hypothetical size of 20001 shares. We also calculate the hourly returns of each stock as the percentage change in the mid-inside quote. Table IV Panel B reports the average liquidity for each hypothetical size and the average return. The numbers are averaged first over time for each stock and then across stocks. To compare the ASX liquidity with liquidity in other markets, we change the unit of liquidity to \$/share by dividing each liquidity by the corresponding hypothetical size, i.e.

$$l_t^{\text{round}} = \int_0^q \left[S_t(Q) - D_t(Q)\right] dQ/q. \tag{7}$$

When $q = 1$, l_t is the inside spread, and when $q > 1$, it is the *ex ante* price impact in dollar per share terms. We can see that liquidity decreases as the hypothetical size increases: the inside spread is less than 3 cents; when the hypothetical order size is 10001 shares, the expected price

[4] We also take hidden orders in the book into consideration. The dataset has an indicator for hidden orders, however the exact hidden number of shares is unknown. We only know that orders need for a dollar volume of at least \$100,000 to qualify as hidden. Thus we add \$100,000/P to the displayed quantity at price P.

impact increases to just over 7 cents at \$0.0702; when we go to 20001 shares along the book, the *ex ante* price impact is the largest at \$0.0993, i.e., just under 10 cents.

To show the intraday dynamics of liquidity, we report the average liquidity hour by hour in Figure 3. Illiquidity as measured by \$/share round-trip transaction cost has a U shape through out the day. This is true for illiquidity measured at the beginning of the book and along the book ($q = 1, 5001, 10001, 15001, 20001$ shares). This result implies that liquidity is lower towards the beginning and the end of the trading day. Previous studies on limit order markets such as Hamao and Hasbrouck (1995) document a similar pattern.

We calculate the hourly order-flow ratios as the ratio of buy volume to sell volume,

$$V_{\text{buy}}/V_{\text{sell}},$$

and the hourly order-type ratios as

$$V_{\text{liquidity-decreasing}}/V_{\text{liquidity-increasing}}$$

where V denotes volume measured in number of shares. The variable $V_{\text{liquidity-decreasing}}$ refers to all shares taken off the book when trades and order cancellations happen. The variable $V_{\text{liquidity-increasing}}$ refers to all shares added to the book when new orders are submitted. An order amendment is treated as an order cancellation plus a simultaneous order submission. Table IV, Panel B reports the means of these two ratios averaged across the 19 stocks. Liquidity decreasing volume approximates liquidity increasing volume at a ratio of 0.97, and buy volume is slightly below sell volume with a ratio of 0.88.

Table IV about here

We also compare liquidity along the book with liquidity at the inside.[5] We choose liquidity at $q = 1, 5001, \ldots, 20001$ for this purpose. The average fraction of times when the five liquidities move in the same direction is only 32.8%, which means that when inside liquidity increases/decreases,

[5] To conserve space, we do not report these results here. They are available from the authors upon request.

more than 2/3 of the time, liquidity beyond the inside changes in the opposite direction. It is in this sense that we need to look beyond the best bid and offer on the book when judging liquidity. We also calculate the average Spearman correlation coefficients between the inside liquidity and the other four outside measures. The correlation ranges from 0.43 to 0.66. These results confirm that looking at liquidity along the book provides more information than simply using the inside best bid and offer as liquidity measures.

5.3 Are order flow and order type correlated?

In much of the discussion above on the difference between return and liquidity commonality, and particularly in our simulations, we have implicitly treated order types and order flows as uncorrelated with each other. While we are not aware of any theoretical reasons to suspect that this is not the case, the true correlation between order types and flows is ultimately an empirical matter. In this section we present evidence to support the validity of our assumption underlying the results.

A straightforward way of examining the relationship between order type and order flow is to compute the unconditional correlation between order type and order flow ratios defined in section 5.2 for each of our sample stocks. Table V Panel A reports the Pearson correlation coefficients along with their p-values between the two ratios for each stock and for the overall sample. The correlations are minimal in magnitude for most stocks. Only four stocks have a correlation coefficient that is significant at the 5% level, all others are insignificant. For the significant correlations, the maximum value is 0.31. For the overall sample, the correlation between order flow and order type is 0.035. These results are supportive of our hypothesis that order flow and order type are not related.

Some might expect that perhaps a conditional correlation exists between order type and order flow. Specifically, it might be the case that the correlation exists in trending markets, e.g., buy orders may tend to be market orders when prices are moving upwards. We check for this possibility by computing the likelihood of order flow and order type co-movement conditional on the contemporaneous return in each stock. An up market is when the stock is experiencing higher than average returns, and a down market is defined as the opposite. The order flow and order type variables are the same ratios used in the previous section. For each stock in each market condition, we calculate the conditional joint probabilities of these two ratios being below or above their stock-specific averages in an up or down market.

Table V Panel B reports the cross-sectional averages of the conditional joint probabilities. As a stock experiences below-average returns, sells outweigh buys (25.74% + 28.36% > 22.79% + 23.11%), but yet, the submissions of market orders and limit orders are close (25.74%+22.79% vs. 28.36% + 23.11%). So there is not much of a correlation between order flows and order types during a down market. Similar results for an up market are also obtained. Buys outweigh sells when returns are above average (26.86%+26.85% > 23.97%+22.32%), yet again, market orders and limit orders are submitted at almost the same likelihood (26.86% + 23.97% vs. 26.85% + 22.32%).

Both the overall correlations between order flow and order type and their conditional joint likelihood on stock returns show that they are indeed different. Because they are different economically and statistically, no wonder they will lead to different consequences.

Table V about here

5.4 Two examples from the Australian Stock Exchange

In this section, we provide two examples of stocks that have very little or no correlation in returns, but have positive correlation in liquidity. The low return correlation make them suitable candidates for the traditional diversification purpose, but the positive correlation in liquidity may make it difficult to actually realize the diversification benefits. The stocks are NCP (News Corporation) and RIO (Rio Tinto Limited) for example 1; PBL (Publishing & Broad) and WPL (Woodside Petroleum) for example 2. As we can see from Table VI, both examples show that the two stocks have low return correlations (about 0.04 with p value >15%), but significant (p values < 0.0001) and relatively large liquidity correlations for both small- and large-size hypothetical trades. The liquidity correlation for both examples stand at around 0.17. Although the liquidity correlation magnitudes are not big, compared to the miniscule return correlations, they are substantial.

Note that we do not argue this will be the case for every pair of stocks or for every portfolio. In fact, there are pairs of sample stocks that have a very high return correlation but not much liquidity correlation. Our argument is that it is *possible* for stocks to have negative or no correlations in returns but positive correlations in liquidity. The two examples above just demonstrate this possibility.

Finally, also reported in Table VI, we use the same order-flow and order-type ratios to calculate order-flow correlations and order-type correlations. We find the results consistent with our statement in Section 4 and the simulation results in the previous section: order flow correlation leads to return correlation, while order type correlation leads to liquidity correlation. In both examples, order flow correlations are close to return correlations, but not in line with liquidity correlations, and order-type correlations are close to liquidity correlations, but not to return correlations. Of course, the simple fact that order-flow correlations approximate return correlations and order-type correlations approximate liquidity correlations for these two examples may not be enough to verify our statement, and to address this concern, we turn to regressions using the entire sample.

Table VI about here

5.5 Regressions of return correlations and liquidity correlations on order-type and order-flow correlations

The purpose of the regression analysis is to examine whether order type correlations and order flow correlations are related to return correlations and liquidity correlations. We build every possible pair of the 19 sample stocks and thus have a sample size of $19 \times 18/2 = 171$ pairs. For each pair, we calculate the return correlation, the liquidity correlations for 21 hypothetical trade sizes ($q = 1, 1001, \ldots, 20001$), as well as their order-type and order-flow correlations based on the hourly ASX order book data. All variables are defined as in previous calculations. We run two types of simple OLS regressions: First, we regress return correlations on order-type correlations and order-flow correlations; then we regress liquidity correlations on order-type correlations and order-flow correlations.

Since the 171 observations are pair-wise correlations, there might be concern that they are not independent, which would influence significance tests. For example, if the correlation between stock x and stock y is 0.8, and the correlation between stock y and stock z is 0.9, then intuitively, the correlation between stock x and stock z could be similarly high. More generally, if the correlation between x and z, ρ_{xz}, depends on ρ_{xy} and ρ_{yz}, then the 171 observations are not independent. However, this is not necessarily a problem in practice, since mathematically, ρ_{xz} is bounded between $\rho_{xy}\rho_{yz} - \sqrt{(1-\rho_{xy}^2)(1-\rho_{yz}^2)}$ and $\rho_{xy}\rho_{yz} + \sqrt{(1-\rho_{xy}^2)(1-\rho_{yz}^2)}$. Given any ρ_{xy} and ρ_{yz}, ρ_{xz} can be virtually anywhere between -1 and 1, the normal boundaries of any correlation. For example, if ρ_{xy} and ρ_{yz} are 0.3 each, then ρ_{xz} is between -0.82 and 1. Therefore, we cannot infer ρ_{xz} from ρ_{xy} and ρ_{yz}, and the concern of dependence among observations should not be a serious one.

Table VII reports the regression results. As we can see from the first set of results, order-flow correlations are strongly and positively related with return correlations: the t-value is 2.36. On the other hand, order-type correlations do not explain return correlations: the coefficient of order-type correlation is insignificant with a t-value of 1.43. The results from this regression confirm that order flows determine returns.

We run the second regression for every hypothetical order size, and Table VII shows that order-type correlation is always positively and significantly related to liquidity correlation for both small and large hypothetical trades. All coefficients are significant at the 5% level. As we move further along the order book, the impact of order-type correlations on liquidity correlations generally becomes stronger (with larger coefficients and higher t values), especially after 10001 shares. For example, for one share, the coefficient on order-type correlation is 0.25 with a t value of 2.10. When we reach the largest size of 20001 shares, the coefficient is 0.37 with a t value of 2.72. Contrary to the significant impact of order-type correlations on liquidity correlations, order-flow correlations do not affect liquidity correlations at all. All the order-flow correlations are insignificant with t

values capped at 0.91. This result shows that order-flow correlations are consistently dominated by order-type correlations in explaining liquidity correlations.[6]

Table VII about here

5.6 Liquidity and liquidity commonality under different market conditions

From the examples of liquidity crises such as the 1987 market crash, it seems that the deterioration of liquidity, or perhaps a systematic liquidity crash, are more prone to occur when the market experiences extreme price movements and investor activities become overwhelmingly imbalanced. To see whether there is such a relationship in our sample, we calculate the likelihood of liquidity movement and liquidity co-movement conditional on extreme market movements. Liquidity is measured along the book for trade sizes of $q = 1, 5001, \ldots, 20001$ shares for each of the 19 stocks in the sample. An extreme up (down) market is defined as a period during which the stock's return fell into the top (bottom) decile of the empirical return distribution.

Table VIII reports the cross sectional average likelihood of liquidity movements and co-movements conditional on extreme up and down markets. There is very little evidence of a systematic difference in the conditional likelihood of liquidity movement between extreme up and down markets. There is, however, some evidence of an asymmetry in liquidity co-movement. For example, at $q = 5001$, the likelihood of both stocks having lower liquidity conditional on both stocks experiencing an extreme down market is 30.82%, while the same likelihood conditional on an extreme up market is only 24.54%. The asymmetry is reversed when we consider liquidity improvements in both stocks. The likelihood conditional on an extreme up market is 16.99% and 14.55% conditional on an extreme down market. Results for other quantities are similar, albeit the asymmetries are not as stark at $q = 1$.

Yet another, and perhaps the most important, asymmetry stands out: Extreme market movements

[6] We also check for the possibility of multicollinearity among the explanatory variables using the Belsley (1980) index and find no multicollinearity exists for the right-hand variables of the regressions. The calculated index is 4.519, which is smaller than even the weak multicollinearity threshold of 5.

tend to deteriorate rather than improve liquidity. This is evidenced both in Table VIII, Panel A and Panel B. Panel A shows that regardless of whether we experience an extreme up or down market, the likelihood that liquidity decreases is often 10% higher than the likelihood that it increases. Using liquidity measured at $q = 5001$ as an example again, the likelihood that liquidity decreases when the market is down is 49.91% while the likelihood that liquidity increases is only 35.29%. Panel B shows that for two stocks in an extreme down market, the likelihood that both liquidity deteriorates is 30.82% while the likelihood for both to increase is only 14.55%. The same result carries over to extreme up markets. Here, the likelihood that both stocks' liquidity decreases is 24.54%, while the likelihood that both improve is only 16.99%. Results for liquidity measured at other points along the book are similar.

The asymmetry between liquidity deterioration and improvement stands out as an important result because it highlights the downward risk of liquidity commonality. Liquidity commonality would be more welcome if it manifested itself by simultaneous increases across stocks, but unfortunately, this is seldom the case. Instead, liquidity commonality reveals itself more through deterioration across stocks. Together with the fact that cross-sectional liquidity decreases more often in a down market than in an up market, the chance for a so-called 'perfect storm' in a financial market does not appear to be all that remote.

6 Concluding Remarks

In this paper, we study the importance of liquidity commonality for investment applications such as portfolio choice and asset allocation by examining the different economic forces underlying return and liquidity co-movement. We argue that liquidity co-movement is caused by cross-sectional correlation in order types, while return co-movement is due to order flow correlations. The two types of co-movement are results of different economic forces in that order type refers to market and limit orders while order flow refers to order direction and size. Market orders always consume liquidity, regardless of their direction and size, while limit order always supply it. On the other hand, buy orders tend to raise price no matter they are market or limit orders, while sell orders tend to lower it.

Following Irvine, Benston and Kandel (2000) and Coppejans, Domowitz and Madhavan (2000), we measure liquidity as a functional of the underlying supply and demand functions. Since supply and demand are most visibly expressed in an open limit-order market, we choose this market structure for both our extensive simulations and our empirical tests. Our simulation results clearly show that return and liquidity commonality are the effects of different economic forces. Our empirical evidence from the Australian Stock Exchange provides further verification of this finding. First, we show that order type and order flow are not highly correlated with each other, which implies that return and liquidity commonality are quite separate phenomena. Next, we highlight two examples of stock pairs that have small return correlations but have significantly positive liquidity correlations. Strikingly, these stocks' order-flow correlations are close to their return correlations but not their liquidity correlations, and their order-type correlations are close to their liquidity correlations but not their return correlations.

Turning to our full sample, regression analyses of return correlations on order-type correlations and order-flow correlations, and of liquidity correlations on order-type correlations and order-flow correlations show that order-flow correlations dominate order-type correlations in explaining return correlations, and order-type correlations dominate order-flow correlations in explaining liquidity correlations. Finally, we document the asymmetry between the likelihood of a deterioration and an improvement in liquidity and liquidity commonality in extreme markets. We find that liquidity commonality manifests itself more often through a common deterioration.

All our results demonstrate that return commonality and liquidity commonality are not caused by the same economic forces: order type determines liquidity and order flow determines return. Therefore, it is possible for stocks to have negative or little return correlations (which makes them ideal for diversification purposes in the Markowitz sense), but strongly positive liquidity correlations. That this possibility is neither only theoretical nor negligible could be witnessed during the 1987 stock market crash and the collapse of LTCM as an outcome of the Russian debt crisis. In a less extreme context, liquidity commonality may potentially hamper the creation/redemption process of some narrowly based ETFs.

Traditional mean-variance portfolio theory focuses on the first and second moments of returns, and assumes no transaction costs. Past research has shown that the first moment of transaction cost, the liquidity level, is a priced factor. Based on our research, we conclude that the second moment of transaction cost also matters in the sense that it may enter as a separate risk that needs to be minimized jointly with portfolio return risk. Liquidity commonality has an adverse impact on diversification benefits because it is difficult to trade a basket of stocks. The main contribution of this paper is that we go beyond merely documenting the existence of liquidity commonality. Instead, we focus on the causes of liquidity commonality and evaluate its implications for investment by demonstrating a linkage between liquidity commonality and return commonality.

We document the potential challenge to traditional diversification strategies posed by liquidity commonality, but leave the severity of the challenge to further study. While we employ a bivariate analysis in deriving our results,an attempt to examine the issue in a multivariate framework is certainly of interest. Also, we do not examine what drives order placement and what determines the shape of the supply and demand curves. These underlying primitives are key to understanding the ultimate reason behind liquidity commonality and return commonality. They can also help with distinguishing between transient and permanent effects. All these require more thorough study in the future.

Appendix A – A Derivation of Equation (6)

In statistics, we use covariance (correlation) to describe the co-movement between two variables. Since we measure liquidity as a function of supply and demand functions, we extend the covariance between two variables to two functionals to describe commonality in liquidity for two stocks. The covariance between two variables is defined as

$$Cov(x_1, x_2) = E\left[(x_1 - E(x_1))(x_2 - E(x_2))\right] = E(x_1 x_2) - E(x_1)E(x_2). \tag{A.1}$$

For a function $g(x_1, x_2, \ldots, x_k) = g(x)$, the Taylor expansion is

$$g(x) = g(\theta) + \sum_{i=1}^{k} g_i'(\theta)(x_i - \theta_i) + O(n^{-1}),$$

where $\theta = E(x)$, i.e., $\theta_i = E(x_i)$, g(x) is differentiable at θ, and $g_i'(\theta) = \partial g(x)/\partial x_i$ evaluated at θ. Thus,

$$E\left[g(x)\right] = g(\theta) + \sum_{i=1}^{k} g_i'(\theta)(E(x_i) - \theta_i) + o(n^{-1}) = g(\theta) + o(n^{-1}), \tag{A.2}$$

and

$$Var\left[g(x)\right] = E\left[g(x) - E(g(x))\right]^2 = E[\sum_{i=1}^{k} g_i'(\theta)(x_i - \theta_i)]^2 + o(n^{-1}) \tag{A.3}$$

$$= \sum_{i=1}^{k} [g_i'(\theta)]^2 var(x_i) + \sum_{i \neq j} \sum_{j=1} k g_i'(\theta) g_i'(\theta) cov(x_i, x_j) + o(n^{-1}) \tag{A.4}$$

Similarly, for two functions g(x) and h(x), if not all the $g_i'(\theta) = 0$ or $h_i'(\theta) = 0$, we have

$$Cov[g(x), h(x)] = \sum_{i=1}^{k} g_i'(\theta) h_i'(\theta) var(x_i) + \sum_{i \neq j} \sum_{j=1}^{k} g_i'(\theta) h_i'(\theta) cov(x_i, x_j) + o(n^{-1}) \tag{A.5}$$

Now, we can generalize (A.5) to two functionals. Since (A.5) is derived from Taylor expansions of $g(x)$ and $h(x)$ and requires the functions $g(x)$ and $h(x)$ to be differentiable, we need to verify that the functional liquidity defined in this paper is differentiable in its arguments S and D and the Taylor expansion for the functional liquidity exists. These can be shown from (A.6) to (A.9). The liquidity measure can be written as a functional $F(y) = \int_0^q f(y(Q))dQ$ with $f(y) = y(Q) = S(Q) - D(Q)$. So,

$$\text{liquidity} = F(y) = \int_0^q y(Q)dQ = \int_0^q [S(Q) - D(Q)]\, dQ. \tag{A.6}$$

When there is a small variation $dy = e\, h(Q)$ in y, where $e \to 0$ and $h(Q)$ is a function of Q defined in the range of $[0, q]$, the change in F is

$$\Delta F = e \int_0^q (\partial f/\partial y)h(Q)dQ + o(e^2) = \int_0^q (\partial f/\partial y)dy + (e^2) \tag{A.7}$$

(A.7) is essentially the functional Taylor expansion of F about the function y. If we assume fixed end-point variations, i.e., $h(0) = 0$ and $h(q) = 0$, then the first functional derivative of the functional $F(y)$ with respect to y is,

$$F'(y) = \Delta F/\Delta y = \partial f/\partial y = 1. \tag{A.8}$$

Therefore the first functional derivatives of the functional liquidity with respect to S and D are

$$l'(S) = \Delta l/\Delta S = \Delta l/\Delta l y \Delta y/\Delta S = 1,$$

and

$$l'(D) = \Delta l/\Delta D = \Delta l/\Delta y \times \Delta y/\Delta D = -1 \tag{A.9}$$

Now, suppose we have two stocks i and j, and their liquidity are l_i and l_j respectively. For given order sizes q_i and q_j, we have

$$l_i = l(S_i, D_i) = f(S_i, D_i, S_j, D_j),$$

where

$$f'(S_j) = f'(D_j) = 0, l'(S_i) = f'(S_i) = 1, l'(D_i) = f'(D_i) = -1,$$

and

$$l_j = l(S_j, D_j) = f(S_j, D_j, S_i, D_i) = g(S_i, D_i, S_j, D_j),$$

where

$$f'(S_i) = f'(D_i) = g'(S_i) = g'(D_i) = 0, l'(S_j) = f'(S_j) = g'(S_j) = 1, l'(D_j) = f'(D_j) = g'(D_j) = -1.$$

Since both the first derivative and the Taylor expansion exist for the functional liquidity, we can now generalize (A.5) to the covariance between two functionals:

$$Cov(l_i, l_j) = Cov(f(x), g(x)),$$

where $x = (x_1 x_2, x_3, x_4) = (S_i, D_i, S_j, D_j)$, and $f_1' = 1, f_2' = -1, f_3' = f_4' = 0; g_3' = 1, g_4' = -1, g_1' = g_2' = 0$;

$$\begin{aligned}
&= \sum_{k=1}^{4} f_k'(\theta) g_k'(\theta) var(x_k) + \sum_{k \neq m} \sum_{m=1}^{4} f_k'(\theta) g_k'(\theta) cov(x_i, x_j) + o(n^{-1}) \\
&= 0 + f_1' g_3' Cov(S_i, S_j) + f_1' g_4' Cov(S_i, D_j) + f_2' g_3' Cov(D_i, S_j) + f_2' g_4' Cov(D_i, D_j) + o(n^{-1}) \\
&= Cov(S_i, S_j) - Cov(S_i, D_j) - Cov(D_i, S_j) + Cov(D_i, D_j) + o(n^{-1})
\end{aligned}$$

When n gets large,

$$Cov(l_i, l_j) = Cov(S_i, S_j) - Cov(S_i, D_j) - Cov(D_i, S_j) + Cov(D_i, D_j).$$

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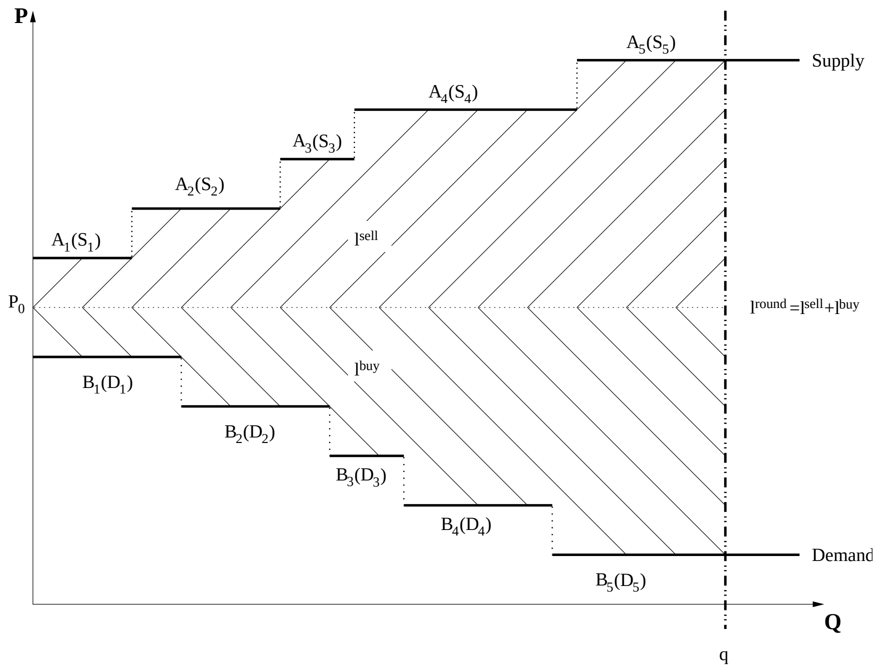

Figure 1: The instantaneous supply and demand step functions and liquidity

A_is are the i^{th} best offer price, and S_is are the corresponding quantity; B_is are the i^{th} best bid price, and D_is are the corresponding quantity. l^{round} is the shaded area between supply and demand; l^{sell} and l^{buy} are respectively the upper half and the lower half of the shaded area separated by the straight line $P_0 = (A_1 + B_1)/2$. q is a hypothetical order size.

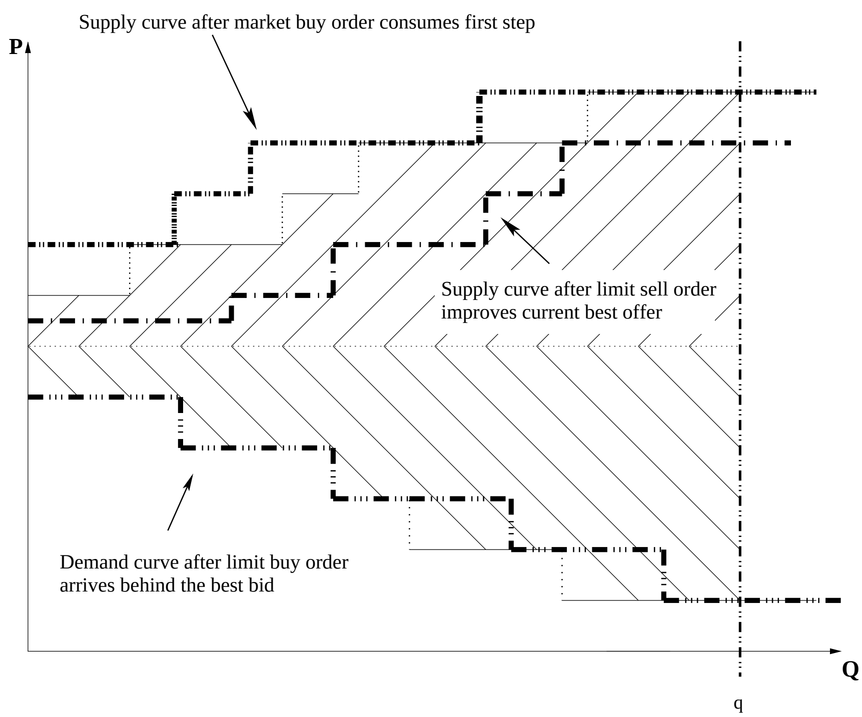

Figure 2: The movement of supply/demand curve.

This figure shows what happens to the supply and demand curves when orders arrive. The original supply and demand curves are shown as a solid line. The original (il)liquidity for a trade size q is represented by the shaded area.

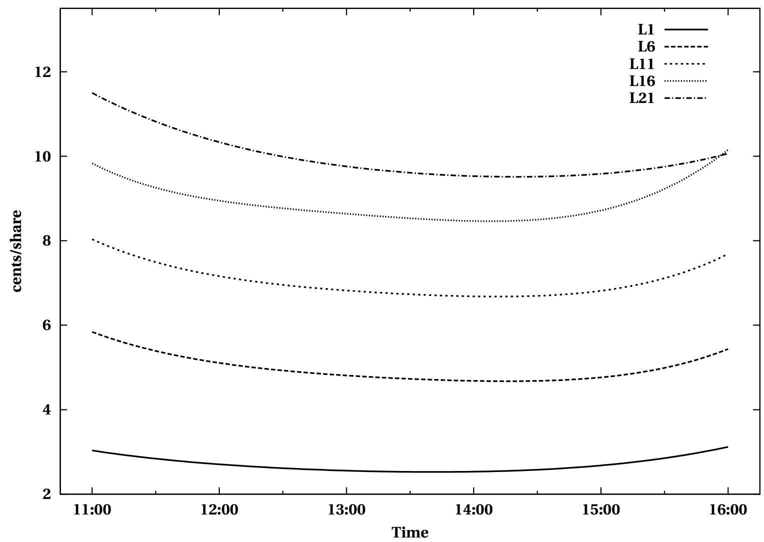

Figure 3: Liquidity throughout the trading day. Shown are smoothed curves of average round-trip costs for various hypothetical trade sizes.

Table I: Simulation I of the limit order book

This table reports the average Spearman correlation coefficients for the liquidity and returns of two stocks from the first set of simulations. Liquidity of each stock is calculated as $l_t^{\text{round}} = \int_0^q [S_t(Q) - D_t(Q)]dQ$ for 10 hypothesized trade sizes $q = 1, 301, 601, \ldots, 2701$ (L1-L10, respectively, in the table). Return is calculated as percentage change in midprice between the best bid and offer prices. This set of simulations randomizes order flows, but control order types. p_1 is the probability that one order is a market order and the other is a limit order. For each stock, 50% of the orders are buys and the rest are sells. Order size follows a uniform distribution between 100 and 3000 and is not restricted to be the same for the two stocks. p_1 is increased from 0.1 to 0.9. 500 simulations are run for every p_1. The reported correlations are averages over the 500 simulations.

p_1	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	Return
0.1	0.4558	0.4511	0.4436	0.4353	0.4224	0.4092	0.3961	0.3840	0.3709	0.3584	−0.0062
0.2	0.3359	0.3321	0.3236	0.3179	0.3111	0.3027	0.2954	0.2856	0.2771	0.2692	−0.0052
0.3	0.2511	0.2475	0.2416	0.2355	0.2281	0.2189	0.2102	0.2012	0.1913	0.1817	−0.0048
0.4	0.1281	0.1267	0.1243	0.1208	0.1194	0.1173	0.1128	0.1099	0.1068	0.1039	−0.0034
0.5	0.0415	0.0403	0.0398	0.0379	0.0391	0.0370	0.0344	0.0311	0.0298	0.0268	−0.0076
0.6	−0.0590	−0.0512	−0.0443	−0.0391	−0.0302	−0.0255	−0.0201	−0.0160	−0.0125	−0.0064	−0.0053
0.7	−0.1493	−0.1507	−0.1502	−0.1509	−0.1492	−0.1460	−0.1423	−0.1402	−0.1343	−0.1296	−0.0024
0.8	−0.2639	−0.2604	−0.2589	−0.2559	−0.2480	−0.2438	−0.2379	−0.2298	−0.2217	−0.2161	−0.0029
0.9	−0.3601	−0.3558	−0.3527	−0.3494	−0.3433	−0.3366	−0.3297	−0.3189	−0.3098	−0.2986	−0.0027

Table II: Simulation II of the limit order book

This table reports the average Spearman correlation coefficients for the liquidity and returns of two stocks from the second set of simulations. Liquidity of each stock is calculated as $l_t^{\text{round}} = \int_0^q [S_t(Q) - D_t(Q)]dQ$ for ten hypothesized trade sizes $q = 1, 301, 601, \ldots, 2701$ (L1-L10, respectively, in the table). Return is calculated as percentage change in midprice between the best bid and offer prices. This set of simulations randomizes order types, but controls order flows. p_2 is the probability that one order is a buy while the other is a sell. The cases of both orders being buys or sells are equally likely with a probability of $(1 - p_2)/2$ each. Order size is uniformly distributed between 100 and 3000 and identical across both stocks. For each stock, half of the orders are market orders and the other half are limit orders. p_2 is increased from 0.1 to 0.9. 500 simulations are run for every p_2. The reported correlations are averages over the 500 simulations.

p_2	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	Return
0.1	−0.0108	−0.0093	−0.0082	−0.0055	−0.0037	−0.0044	−0.0062	−0.0072	−0.0083	−0.0091	0.2144
0.2	−0.0293	−0.0297	−0.0280	−0.0256	−0.0262	−0.0295	−0.0290	−0.0315	−0.0295	−0.0306	0.1565
0.3	−0.0357	−0.0334	−0.0325	−0.0335	−0.0344	−0.0347	−0.0339	−0.0342	−0.0317	−0.0322	0.1079
0.4	−0.0471	−0.0476	−0.0484	−0.0487	−0.0505	−0.0495	−0.0488	−0.0501	−0.0473	−0.0447	0.0487
0.5	−0.0352	−0.0384	−0.0407	−0.0410	−0.0429	−0.0453	−0.0447	−0.0428	−0.0444	−0.0413	−0.0021
0.6	−0.0366	−0.0373	−0.0375	−0.0375	−0.0367	−0.0347	−0.0340	−0.0317	−0.0291	−0.0260	−0.0544
0.7	−0.0298	−0.0283	−0.0290	−0.0281	−0.0266	−0.0264	−0.0271	−0.0272	−0.0250	−0.0227	−0.1100
0.8	−0.0017	−0.0019	−0.0007	0.0008	0.0013	0.0020	0.0032	0.0025	0.0049	0.0048	−0.1633
0.9	−0.0002	−0.0008	−0.0011	−0.0014	−0.0012	0.0018	0.0003	−0.0011	−0.0022	−0.0024	−0.2106

Table III: Simulation III of the limit order book

This table reports the average Spearman correlation coefficients for the liquidity and returns of two stocks from the third set of simulations. Liquidity of each stock is calculated as $l_t^{\text{round}} = \int_0^q [S_t(Q) - D_t(Q)]dQ$ for 10 hypothesized trade sizes $q = 1, 301, 601, \ldots, 2701$ (L1-L10, respectively, in the table). Return is calculated as percentage change in midprice between the best bid and offer prices. This set of simulations controls both order types and order flows. p_3 is the probability that one order is a market order and the other is a limit order with both orders having the same direction. Let $1 - p_3$ denote the probability that both orders are of the same type and opposite direction. Order size is uniformly distributed between 100 and 3000 and identical across both stocks. p_3 is increased from 0.1 to 0.9. 500 simulations are run for every p_3. The reported correlations are averages over the 500 simulations.

p_3	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	Return
0.1	0.6887	0.6857	0.6820	0.6778	0.6734	0.6696	0.6648	0.6598	0.6557	0.6508	−0.4829
0.2	0.5280	0.5244	0.5207	0.5164	0.5125	0.5072	0.5020	0.4968	0.4917	0.4862	−0.2913
0.3	0.3710	0.3680	0.3646	0.3613	0.3565	0.3513	0.3461	0.3400	0.3344	0.3293	−0.1757
0.4	0.1241	0.1230	0.1212	0.1201	0.1180	0.1159	0.1140	0.1129	0.1117	0.1111	−0.0851
0.5	−0.0652	−0.0660	−0.0660	−0.0643	−0.0648	−0.0654	−0.0656	−0.0649	−0.0646	−0.0638	−0.0237
0.6	−0.1348	−0.1366	−0.1397	−0.1421	−0.1441	−0.1452	−0.1468	−0.1484	−0.1499	−0.1500	0.0327
0.7	−0.3372	−0.3365	−0.3364	−0.3358	−0.3357	−0.3346	−0.3327	−0.3305	−0.3280	−0.3269	0.0774
0.8	−0.4542	−0.4543	−0.4525	−0.4516	−0.4526	−0.4528	−0.4522	−0.4529	−0.4526	−0.4521	0.1347
0.9	−0.5607	−0.5591	−0.5595	−0.5617	−0.5629	−0.5627	−0.5638	−0.5661	−0.5654	−0.5673	0.1814

Table IV: Summary Statistics of ASX Stocks

This table reports the summary statistics of the 19 stocks that consistently make up the ASX 20 index during 3/1/2000 to 12/31/2000. Variables are aggregated to an hourly frequency from 11:00am to 4:00pm. Panel A reports the average distribution of order size and trade size (in number of shares). Panel B reports the average return, liquidity, order-flow ratio and order-type ratio. Return is calculated as the percentage change in midprice between the best bid and offer prices. Liquidity is calculated as $\int_0^q [S_t(Q) - D_t(Q)]dQ/q$ for 11 hypothetical trade sizes $q = 1, 2001, \ldots, 20001$ shares. Order flow ratio is the ratio between buy volume and sell volume. Order type ratio is the ratio between liquidity-decreasing volume and liquidity-increasing volume. Volume is in number of shares.

Panel A: Order Size and Trade Size					
Percentile	10%	25%	50%	75%	90%
Order size	197	600	2111	7500	17838
Trade size	176	500	1204	4003	10000

Panel B: Average Returns, Liquidity, Order-flow Ratios and Order-type Ratios	
Return	7.95E-5
Liquidity	
1	0.0269
2001	0.0362
4001	0.0451
6001	0.0537
8001	0.0625
10001	0.0702
12001	0.0786
14001	0.0855
16001	0.0891
18001	0.0946
20001	0.0993
Order-flow ratio	0.88
Order-type ratio	0.97

Table V: Relationship between Order Flow and Order Type Ratios

This table reports the relationship between order flow ratios and order type ratios. Panel A reports the unconditional Pearson correlation coefficients between the two ratios for each stock. Panel B reports the cross-sectional average conditional joint probabilities of the two ratios being below or above their stock-specific averages on an up or down market. An up (down) market is when the stock is experiencing higher (lower) than average returns. Order flow ratio is the ratio between buy volume and sell volume. Order type ratio is the ratio between liquidity-decreasing volume and liquidity-increasing volume. Volume is in number of shares.

Panel A		
Stock	Correlation	p-value
1	−0.001	0.9599
2	0.073	0.0098
3	0.026	0.3571
4	0.050	0.0831
5	0.310	<.0001
6	−0.011	0.6996
7	0.023	0.4098
8	0.014	0.6272
9	−0.018	0.5204
10	0.131	<.0001
11	0.058	0.0385
12	0.005	0.8556
13	−0.009	0.7539
14	−0.003	0.9143
15	0.024	0.3953
16	0.030	0.2917
17	−0.005	0.8533
18	0.002	0.9418
19	0.003	0.9242
Overall	0.035	<.0001

Panel B			
Buy/Sell ratio	Market/Limit ratio	Market	Conditional Probability
low	low	down	25.74%
low	high	down	28.36%
high	low	down	22.79%
high	high	down	23.11%
low	low	up	23.97%
low	high	up	22.32%
high	low	up	26.86%
high	high	up	26.85%

Table VI: Two Examples From the Australian Stock Exchange

This table shows two examples from the Australian Stock Exchange data from 3/1/2000 to 12/31/2000. Example 1 is about stock NCP (News Corporation) and RIO (Rio Tinto Limited); example 2 is about stock PBL (Publishing & Broad) and WPL (Woodside Petroleum). Variables are aggregated hourly from 11:00 am to 4:00pm. Return is calculated as percentage change in the mid-inside-quote. Order flow is calculated as the ratio between buy volume and sell volume. Liquidity is calculated as: $\int_0^q [S_t(Q) - D_t(Q)]dQ$ for 21 hypothetical sizes from 1, 1001, 2001 to 20001 shares. Order type is calculated as the ratio between liquidity-decreasing volume and the liquidity-increasing volume. Volume is in number of shares. Reported in each example are the Spearman correlations and their P-values between the two stocks' returns, order flows, liquidity and order types.

	Example 1: NCP and RIO		Example 2: PBL and WPL	
	Correlation	P-value	Correlation	P-value
Return	0.037	0.1881	0.039	0.1616
Order flow	-0.008	0.7955	0.020	0.4801
Liquidity				
1	0.132	<0.0001	0.172	<0.0001
1001	0.139	<0.0001	0.192	<0.0001
2001	0.161	<0.0001	0.184	<0.0001
3001	0.171	<0.0001	0.177	<0.0001
4001	0.175	<0.0001	0.166	<0.0001
5001	0.175	<0.0001	0.155	<0.0001
6001	0.181	<0.0001	0.157	<0.0001
7001	0.181	<0.0001	0.153	<0.0001
8001	0.181	<0.0001	0.150	<0.0001
9001	0.178	<0.0001	0.156	<0.0001
10001	0.176	<0.0001	0.164	<0.0001
11001	0.178	<0.0001	0.171	<0.0001
12001	0.180	<0.0001	0.181	<0.0001
13001	0.183	<0.0001	0.183	<0.0001
14001	0.183	<0.0001	0.179	<0.0001
15001	0.180	<0.0001	0.173	<0.0001
16001	0.187	<0.0001	0.178	<0.0001
17001	0.176	<0.0001	0.178	<0.0001
18001	0.175	<0.0001	0.178	<0.0001
19001	0.170	<0.0001	0.167	<0.0001
20001	0.173	<0.0001	0.177	<0.0001
Order type	0.133	<0.0001	0.161	<0.0001

Table VII: Regressions of Return Correlations and Liquidity Correlations On Order Type Correlations and Order flow Correlations

This table reports the OLS regression results for the 19 stocks that consistently make up the ASX 20 index from 3/1/2000 to 12/31/2000. All the variables are pair-wise Spearman correlations based on hourly data from 11:00 am to 4:00pm. The number of observation for each regression is 171. Order flow is calculated as the ratio between buy volume and sell volume. Liquidity is calculated as $\int_0^q [S_t(Q) - D_t(Q)]dQ$ for 21 hypothetical sizes from 1, 1001, 2001 to 20001 shares. Order type is calculated as the ratio between liquidity-decreasing volume and the liquidity-increasing volume. Volume is in number of shares. The t values are based on White-corrected standard errors.

	Orderflow corr	t-value	Ordertype corr	t-value	R^2 (in %)
Return correlation	0.73	2.36	0.46	1.43	16.95
Liquidity correlation					
1	0.12	0.91	0.25	2.10	4.68
1001	0.00	0.00	0.27	2.07	4.57
2001	-0.00	-0.00	0.28	2.27	5.21
3001	0.01	0.07	0.29	2.34	5.57
4001	0.04	0.29	0.27	2.14	4.86
5001	0.05	0.36	0.25	2.01	4.20
6001	0.04	0.27	0.26	2.03	4.25
7001	0.04	0.27	0.27	2.08	4.36
8001	0.02	0.18	0.27	2.15	4.45
9001	0.04	0.26	0.28	2.19	4.55
10001	0.04	0.31	0.29	2.26	4.78
11001	0.03	0.25	0.31	2.37	5.20
12001	0.04	0.29	0.32	2.47	5.57
13001	0.03	0.25	0.34	2.61	6.09
14001	0.04	0.28	0.35	2.67	6.25
15001	0.05	0.33	0.36	2.74	6.50
16001	0.03	0.20	0.38	2.83	6.92
17001	0.05	0.38	0.39	2.89	7.38
18001	0.05	0.33	0.38	2.80	7.01
19001	0.08	0.52	0.36	2.68	6.64
20001	0.08	0.55	0.37	2.72	6.81

Table VIII: Asymmetries in liquidity and liquidity commonality between extreme up and down markets

Panel A reports the likelihood of liquidity movement conditional on extreme returns. Panel B reports the likelihood of liquidity co-movement conditional on extreme returns. Liquidity is measured along the book for trade sizes $q = 1, 5001, \ldots, 20001$ shares for each of the 19 stocks in the sample. Liquidity commonality is measured for each possible pair of the 19 stocks ($19 \times 18/2 = 171$ observations). Extreme up (down) markets are defined as periods with a return in the top (bottom) decile of the empirical return distribution.

		Quantity q				
Liquidity	Market Condition	1	5001	10001	15001	20001
Panel A: Likelihood of Liquidity Movement Conditional on Extreme Return						
Deteriorates	Down	38.03%	49.91%	49.89%	49.92%	49.87%
Improves	Down	28.41%	35.29%	35.82%	37.51%	37.55%
Deteriorates	Up	37.09%	47.19%	47.81%	48.30%	47.26%
Improves	Up	28.51%	37.76%	38.29%	39.80%	38.72%
Panel B: Likelihood of Liquidity Co-Movement Conditional on Extreme Returns						
Both deteriorate	Both down	17.00%	30.82%	33.65%	33.87%	32.16%
Both improve	Both down	10.58%	14.55%	15.61%	16.17%	15.51%
Both deteriorate	Both up	15.53%	24.54%	27.82%	27.63%	26.72%
Both improve	Both up	12.03%	16.99%	17.88%	19.20%	17.89%