

Where Risk Control Meets Cost Control in Analytics Development*

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In a portfolio transition, cost-minimizing trading strategies may have the unintended consequence of increasing risk along several dimensions. On the other hand, strategies designed to mitigate risk can involve unacceptable levels of cost, may be sub-optimal once transaction costs are taken into account, or may simply be infeasible.

We suggest that a structured program of measurement, analysis, and control in trading applications involves the convergence of execution tools and portfolio management analytics and techniques. This convergence is a natural result of balancing multiple objectives throughout the trading process. In addition to the tradeoffs involving market impact and opportunity cost, control of tracking error, and the risk of the residual trade list enter the calculation. To those issues, one adds constraints involving benchmark concentration and sector exposures, for example. Finally, there is some urgency to produce a realized set of trades that results in a portfolio that behaves like the target portfolio as quickly as possible, dictating an important time dimension.

Rather than attempt to make this argument in the abstract, we present three examples of portfolio trading tool construction, in which such convergence plays an essential part. The basic concepts are introduced through a brief discussion of trading oriented towards cost control. There, risk models play a role in characterizing opportunity cost and provide a basis for mean-variance optimization leading to coherent strategies. We then expand the portfolio problem to include not only time, but also holdings' tracking error relative to benchmark as the trade evolves. The idea is illustrated by the construction of waves of trades that are consistent with a generalization of the typical efficient frontier. Finally, we turn to portfolio composition, touching upon shifts in the frontier due to the inclusion of transaction costs, and introducing the latter as a means to differentiate between portfolios that are equivalent with the margin of input errors.

Trading for Cost Control

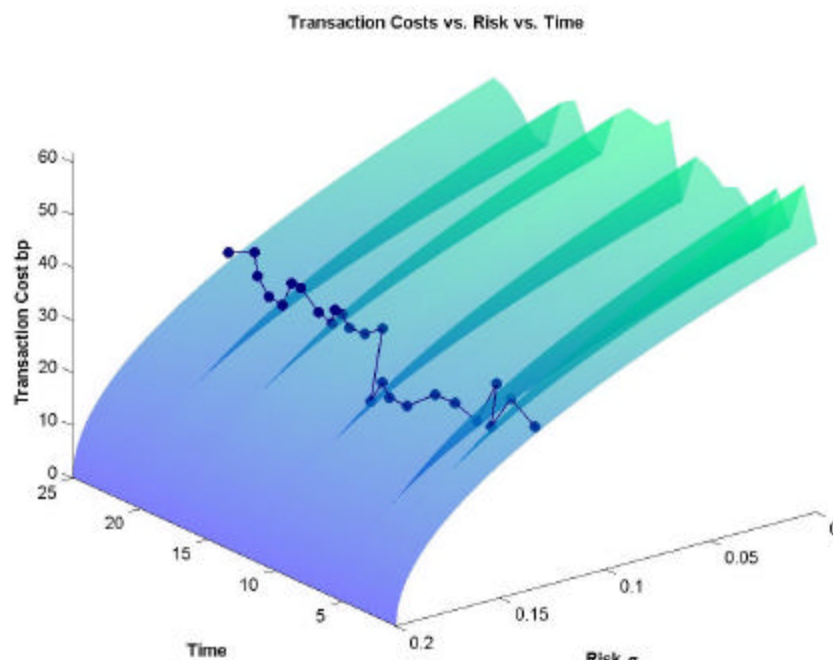
Cost minimizing strategies begin with estimates of price impact costs, and the evaluation of alternatives against pre-trade cost benchmarks. Such

evaluation, and the translation of the analysis into a strategy that optimizes cost of trading, require consideration of opportunity cost. Choice of a trading horizon also requires balancing market impact and opportunity costs, with the former declining over time as the latter increases.

A better, or worse, price may be obtained by waiting for all or part of an execution. The opportunity cost involved in that decision is easy to recognize, but notoriously hard to define quantitatively. When considering trade sizes for which it is reasonable to believe that the market in a stock moves externally to one's own trading, opportunity cost finds representation in terms of the underlying return of the stock. In that case, opportunity cost is viewed as a risk, and defined as the variance or standard deviation of expected returns. A covariance-based model, similar in structure to the multifactor risk models used by asset managers, can be used to predict this cost for the purpose of optimization.¹

One advantage to this approach is that any trade list is treated as though it were a long/short portfolio. Opportunity costs associated with the trade list depend on the covariance of returns. Standard mean-variance optimization techniques can be used to find a balance between expected cost and the risk of the residual trade list, during the execution process. Conceptually, the tradeoff between cost and residual risk forms an efficient frontier, illustrated in Figure 1.

FIGURE 1



In Figure 1, each slice across the time axis will be a typical efficient frontier, albeit now defined in terms of expected cost as opposed to expected return. At each point in time, considerations relevant to the trader and instructions from the portfolio manager define an optimal risk tradeoff, delineating a point along that frontier. The time-series of these choices is represented by the series of connected points along the efficient surface, which is itself constructed from the time-series of efficient frontiers.

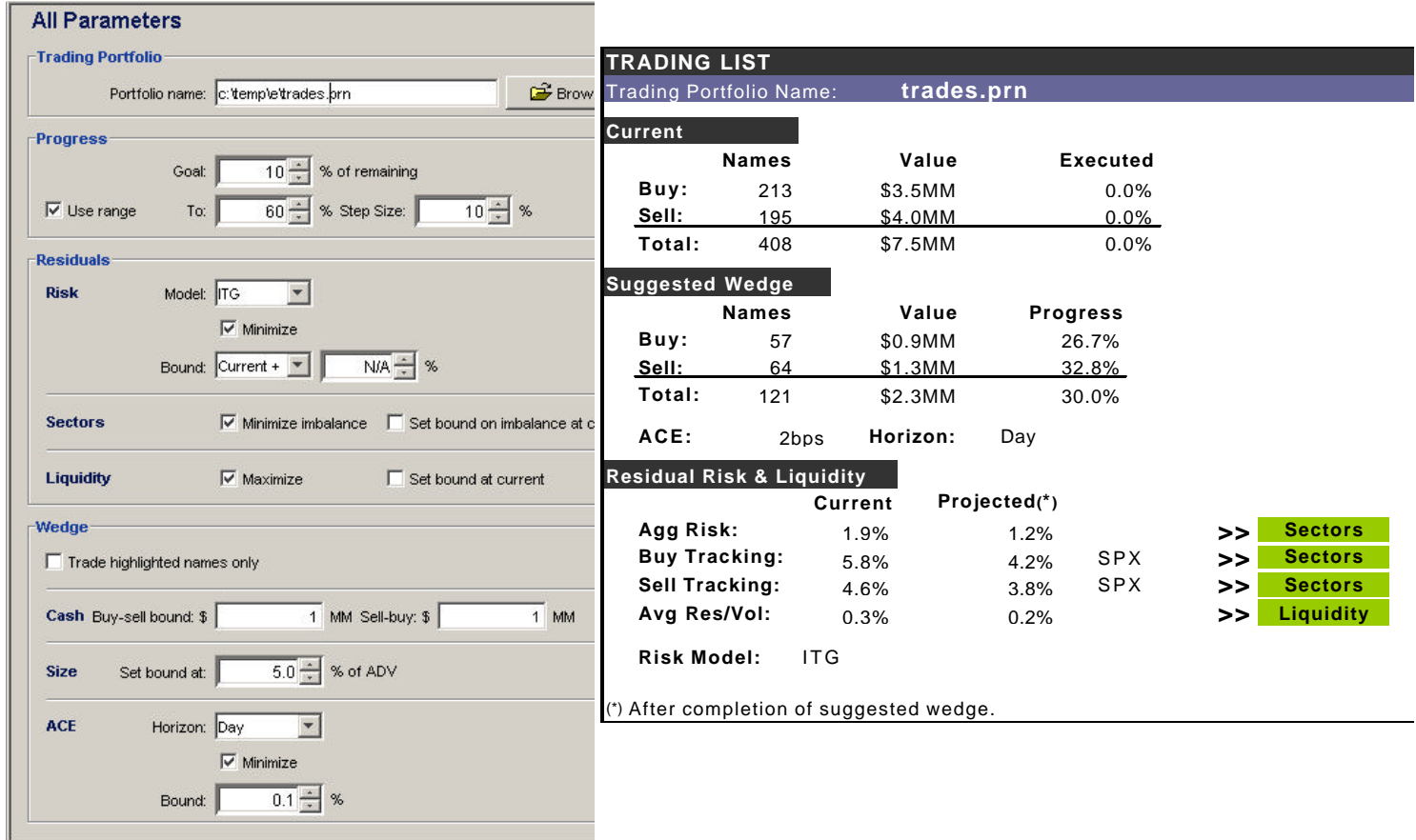
Using dynamic stochastic optimization, this optimal path can be constructed from inputs of expected market impact, covariance of returns, and risk aversion, the latter now relating to the urgency level of the trade. The optimal path is defined by a series of trade waves, each of which is the output of the optimization process.

Expanding the Tradeoffs: List Characteristics and Execution

In order to avoid “optimality limbo” for the asset manager’s portfolio over an extended trading horizon, the concept of an efficient frontier is generalized beyond Figure 1. The objective function in an otherwise classical Markowitz portfolio problem is expanded to include not only time, but also the holdings’ tracking error relative to benchmark, as well as applicable portfolio characteristics constraints. The natural result of this conceptual approach is the ability to construct waves of trades over the horizon, in such a way as to permit statements such as, “no wave that completes 15% of the trade list, while costing 25 basis points, will result in a tracking error lower than 7.8%.” Recognition of such tradeoffs is essential in the decision process leading to successful implementation strategies.

In practice, the trick is to implement the conceptual frontier. An example of a tool used to construct optimal waves from a trade list is illustrated in Figure 2.

FIGURE 2



The user has specified that a series of waves be constructed, each wave representing between 10% and 60% of the value of the entire trade list. The risk, as evaluated by a multi-factor covariance model, and the sector imbalance of the residual trade list, will be minimized. Liquidity of the residual portfolio will be maximized at each step in the process.² The residuals will be cash-balanced within \$1 million. No trade in the wave will represent more than 5% of average daily volume, and the estimated market impact of the wave will be no more than 0.1% of the list's value. In practice, all such choices are at the trader's discretion, and may also reflect instructions from the asset manager.

For illustration, the right hand panel of Figure 2 contains the forecasted results of trading an optimized wave of 30% of the list's value. The risk of the residual portfolio has been reduced, as has the tracking error of each side relative to the S&P 500. Additionally, liquidity has improved, while the estimated

trading cost for this wave of executions is two basis points. The wave can now be traded in response to current market conditions. The optimization process can be repeated throughout the duration of the trade of the portfolio, updated with real-time market data. The result of these series of optimizations effectively defines the opportunities discussed in the context of Figure 1.

Portfolio Composition and Transaction Costs

The presentation thus far has concentrated on trading. The convergence of execution and portfolio tools is not directed solely towards the execution process, however. We provide two examples relating to portfolio construction in this section. The first touches upon a familiar theme, pertaining to shifts in the efficient frontier due to the inclusion of transaction costs in the optimization process. The second example introduces the notion of equivalent portfolios, and the potential use of execution analytics to differentiate between them.

Transaction Costs and Portfolio Construction

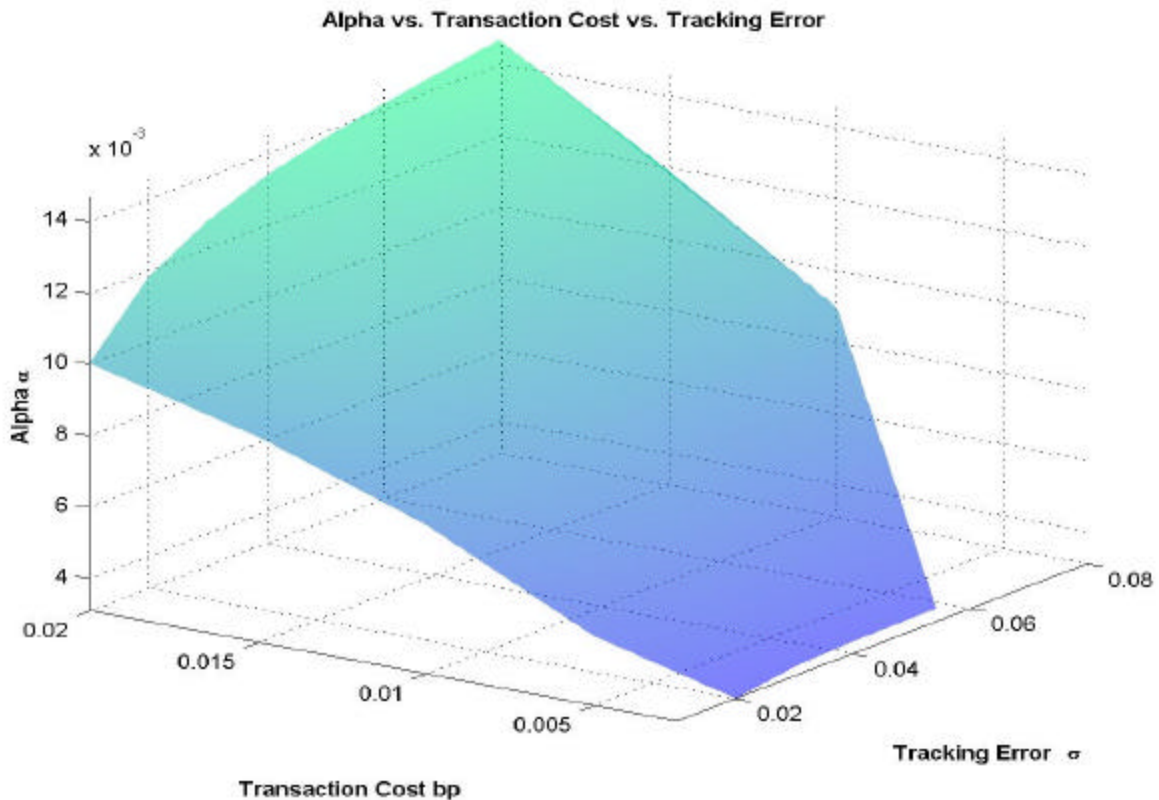
Execution costs vary systematically with factors that are relevant for investment strategy, a principle underlying our previous discussion. Conversely, the estimated mean-variance efficient frontier, hence any perceived gain from diversification, also changes as execution costs are factored into return computations. The size and pattern of a shift in the frontier are determined from two sources. The first relates to the nonlinearity of transaction costs themselves, as returns are adjusted. The second is generated from changes in the correlation structure of returns, as costs are spread non-uniformly across securities. Such movements are compounded as turnover increases.

These relationships usually are illustrated simply by comparing frontiers with and without adjustments for cost. Some additional insight may be gained by graphing cost explicitly, however.

In Figure 3, we plot a more classical return/risk efficient frontier, for the rebalance of a portfolio while considering transaction costs. As we constrain the transaction cost incurred by the rebalance to be smaller, the available set of feasible solutions becomes smaller as well. The potential alpha at any level of risk is less than it would be if transaction costs are allowed to be greater.

Conversely, the lowest level of risk for a given alpha is higher than it would be if transaction costs were allowed to be greater.

FIGURE 3



When rebalancing a portfolio, a portfolio manager not only should choose a level of risk aversion, which represents his or her investment preferences, but also a level of *transaction cost aversion*, representing preferences with respect to urgency of execution. These choices isolate a point on the surface of Figure 3. Because of the non-linearity of transaction costs, one cannot simply isolate a “net alpha.” The surface of Figure 3 must be derived from a series of optimizations, and an investment decision made to choose a point upon it.

Previous studies have shown the effects of cost on portfolio choice to be potentially significant. For example, in the context of a global portfolio, calculations suggest downward movement in the tangency portfolio returns of about 70 basis points. Such a shift is coupled with an increase in volatility by as

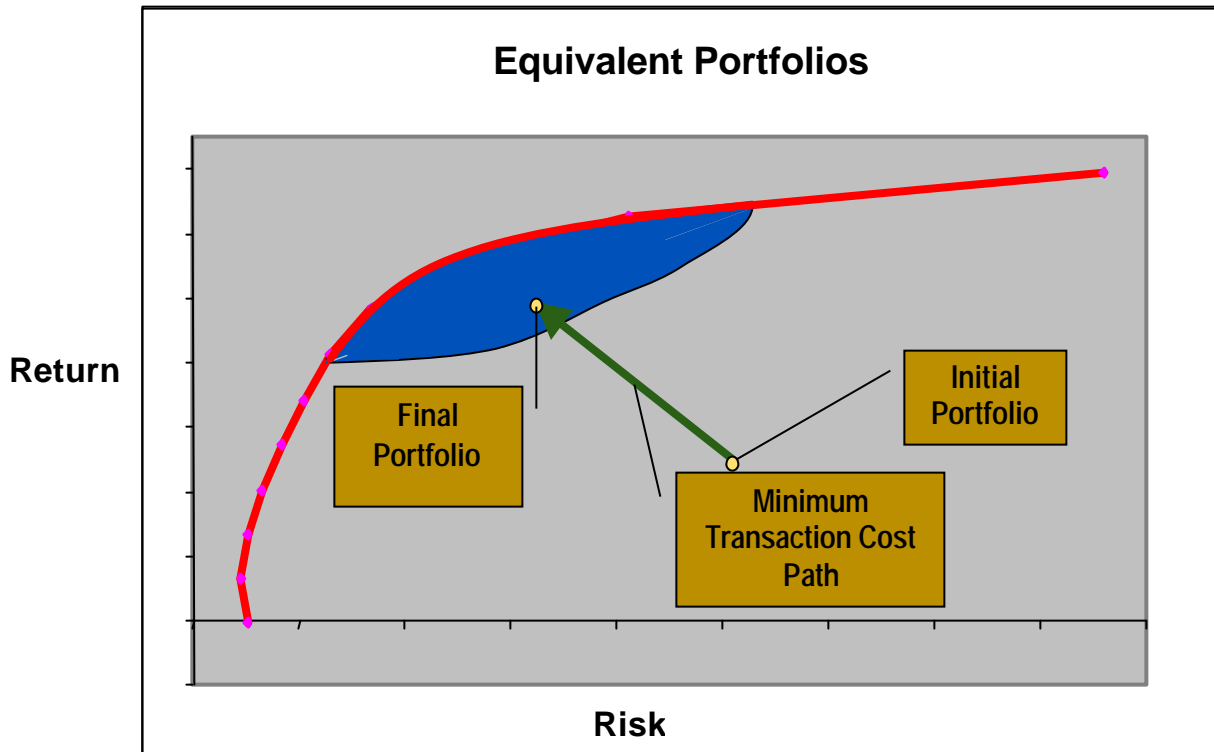
much as 60 basis points, once costs due to annual rebalancing are taken into account. Transaction costs also may generate large moves in the tangency portfolio towards lower cost regions. In the late 1990's, for example, such analysis would have implied an increase in the weight for relatively low-cost Western European markets by 15 percent under semi-annual rebalancing, while the weight for North American securities declines by as much as 36 percent.³

Transaction Costs and Uncertain Inputs

Various studies have documented the effects of changes in inputs on the composition of optimal portfolios, and discussed the related issue of the impact of errors in inputs on investors' utility functions.⁴ We approach this topic somewhat differently, beginning by suggesting that optimal portfolio choices are not necessarily unique in practice. Rather, there are sets of portfolio choices that are equivalent, and the composition of "optimal" portfolios in such sets may be quite different.

The phenomenon of equivalent portfolios arises for (at least) two reasons. Technically, in our experience, the objective function near the optimality point may be very flat. From a practical perspective, this means that there will be many portfolios with nearly identical risk/reward tradeoffs. Second, the inputs to portfolio optimization are measured with error. Regardless of contrary opinions with respect to the first point, the uncertainty problem is universally acknowledged, and the issue is perhaps most obvious in the case of alpha estimates. We therefore concentrate on this case, which is illustrated in Figure 4.

FIGURE 4



Stochastic optimization techniques may be used to identify what might be called an “indifference set” of portfolios. The dark area beneath the efficient frontier consists of portfolios that are equivalent in terms of risk and return within the margin of error for the alpha component of the portfolio.⁵

Starting from the initial portfolio, we attempt to reach a portfolio with characteristics placing it inside the indifference region. Which portfolio should be chosen, since there are practically infinitely many which fit the above criterion?

The answer is to take the path which incurs the least transaction cost. We can think of this as a three-stage problem. First, a set of indifference regions is constructed. This process essentially transforms expected return and covariance information, and the errors inherent in those inputs, into a “fuzzy” frontier. Within this generalized frontier will be areas, in which a given level of risk is consistent with a range of alpha, within the bounds of a statistical confidence interval. Second, a single region within the generalized frontier can be isolated, based on the desired risk/return tradeoff on the part of the manager. Finally, transaction

costs associated with the constraints of moving the initial portfolio into the chosen region are minimized. The result is a portfolio, which is not only optimal within the margin of input error, but also is the least costly to trade into.

Pieces of the Convergence Puzzle

The divide between portfolio creation and optimization, and trading strategy and execution, is often debated and costly to maintain. Good communication between portfolio manager and trader is a key to bridging that gap. Analytical tools that leverage the value inherent on both sides of the divide can go a long way towards enabling that communication.

At the level of analytics development, such convergence involves a variety of factors. Transaction cost analysis is married to portfolio benchmarking. Classical portfolio optimization tools are paired with stochastic dynamic optimization techniques. Ex ante risk and cost projections flow into ex post attribution and performance studies. Finally, measurement of both trading cost and associated risk must contend with the necessity of factoring trading strategy into the equation.

We have touched on only a few of these pieces through the examples presented here. Nevertheless, the conceptual framework provides its own discipline for list trading and associated portfolio composition and management. It is often noted that trading costs and a choice of execution strategy are closely linked, and depend on circumstances surrounding a particular order. Implementation challenges now focus on the integration of portfolio characteristics with order, cost, and strategy, in such a way as to balance risk and net return over the trading cycle.

Endnotes

¹ Factors and factor loadings, however, should be tuned to the short run problem faced by the trader. Our own research suggests that risk models built for daily risk analysis, as opposed to the standard monthly time frame, are superior for this type of work, embodying very different forecasts and risk decomposition.

² Liquidity here is calculated in terms of the average share of volume; other measures could, of course, be used.

³ These examples are taken from “Liquidity, Volatility, and Equity Trading Costs Across Countries and Over Time,” by Ian Domowitz, Jack Glen, and Ananth Madhavan, *International Finance* vol. 4, no. 2, 2001.

⁴ See, for example, V.K. Chopra, “Mean-Variance Revisited: Near Optimal Portfolios and Sensitivity to Input Variations,” *Journal of Investing*, 1993, and V.K. Chopra and W.T. Ziemba, “The Effects of Errors in Means, Variances, and Covariances on Optimal Portfolio Choice,” *Journal of Portfolio Management*, 1993.

⁵ We have exaggerated the size of such a set in the figure to ease presentation. Actual areas are computed based on small triangles used to approximate a confidence region for the efficient frontier.